Consumption Externalities, Production Externalities, and Long-run Macroeconomic Efficiency*

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Abstract

We analyze the effects of consumption and production externalities on capital accumulation. We show that the importance of consumption externalities depends upon the elasticity of labor supply. If the labor supply is inelastic, consumption externalities cause no long-run distortions. Whether there are distortions along the transitional path depends upon consumer preferences. The effects of production externalities are more pervasive; they exert long-run distortionary effects irrespective of labor supply. The optimal taxation to correct for the distortions created by the externalities is characterized. We analyze both stationary and endogenously growing economies, and while there are many parallels in how externalities impact, there are also important differences.

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1. Introduction

Externalities are a fundamental aspect of a modern interdependent economy. The fact that agents interact with one another makes it inevitable that their decisions will influence one another directly, in addition to any indirect impact that may occur through the market place. Economists have long been aware of externalities, and their role has been widely studied in many contexts.[1]

Broadly speaking, we can categorize externalities as (i) consumption externalities, and (ii) production externalities. Recently, the former have been extensively studied in the context of models of jealousy and "keeping up with the Joneses." For example, Abel (1990) and Campbell and Cochrane (1999) studied the effect of consumption externalities on asset pricing and the equity premium. Gali (1994) showed that incorporating keeping up with the Joneses in the preferences can have the same effect as increasing the degree of risk aversion in its implications for asset pricing. Ljungqvist and Uhlig (2000) analyzed the impact of consumption externalities on the effect of short-run macroeconomic stabilization policy. Dupor and Liu (2003) defined different forms of consumption externalities and explored their relationship with equilibrium over-consumption. These models typically either are pure consumption models that take output as given, or alternatively, adopt a simple production technology in which output depends only upon labor inputs.

In contrast, production externalities provided the cornerstone of the endogenous growth model pioneered by Romer (1986).[2] The key feature of this literature is that even though the individual firm's capital stock may be subject to diminishing marginal physical product, the presence of an aggregate production externality enhances its productivity so that in equilibrium the economy is able to sustain a steady growth rate. Typically, the externality is specified by the presence of the aggregate capital stock in individual production functions, which serves as a proxy for knowledge, as in Romer's original contribution, or by the presence of productive government expenditure, as in Barro (1990) and Turnovsky (1996).[3]

Empirical evidence on the importance of externalities is sparse and indirect, but overall, the

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[1] The earliest discussion of productive externalities dates back to Marshall (1890). Consumption externalities were emphasized in early work by Veblen (1912) and were first formalized as a determinant of aggregate consumption by Duesenberry (1949) in his development of the "relative income hypothesis".

[2] Actually the type of externality proposed by Romer can be found in an early formulation by Frankel (1962).

[3] Several models, including another seminal paper, Romer (1990), introduce knowledge explicitly, rather than as a proxy.

existing evidence provides convincing support for the importance of consumption externalities. Constantinides (1990) showed that habit formation provides a solution to the equity premium puzzle. In his study of asset pricing, Abel (1990) introduced a reference consumption level into utility that depended upon a weighted average of the agent's and the economy-wide average of immediate past consumption and finds that the model calibrates most successfully if the weight is fully assigned to the aggregate, so that utility depends upon relative consumption. Easterlin (1995) discusses a number of studies relating happiness to income growth. Based on various time series studies employing US, European, and Japanese data, he concludes that the evidence supports the proposition that raising income of all does not increase everyone's happiness, again implying the presence of negative externalities. Clark and Oswald (1996) present some direct empirical evidence for British workers, showing that their reported satisfaction levels are inversely related to their comparison wage rates. Frank (1997) provides a comprehensive discussion based on both the psychological evidence and the more fragmentary evidence in behavioral economics. He concludes that both these sources support the claim that satisfaction depends upon the agent's relative position, again emphasizing the role of the environment and the externalities it generates.

The evidence on production externalities, while less conclusive, is nevertheless also quite compelling. Caballero and Lyons (1990, 1992) analyze externalities within the context of EU and US manufacturing industries. Although they find evidence of externalities, their analysis has been questioned by Basu and Fernald (1996) and Burnside (1995). On the other hand, using a methodology similar to that employed by Basu and Fernald, Benarroch (1997) finds evidence of externalities at the two-digit industry level in Canada. Finally, the empirical evidence initiated by Aschauer (1989) suggesting how public capital enhances private productivity can be taken as being further evidence of the role of production externalities.

In the light of the evidence, the question of how these two types of externalities affect economic performance is important. To what extent do they introduce distortions into the savings and investment decisions, and if so, what are the appropriate corrective policy responses? This issue has been partially addressed in the context of the basic Romer (1986) model, where the production externality has been shown to lead to over-consumption and sub-optimally low growth. But it has not been considered in the context of the Ramsey (1928) model, and the role of consumption externalities on the allocation of capital has received little systematic discussion.[4] Yet it is clearly evident that consumption externalities, with their impact on agents' savings-consumption decisions, can have significant consequences for the economy's capital stock. In addition, the two types of externalities are likely to have compounding effects. Consumption externalities can be expected to have consequences for production externalities, and vice versa.

In this paper we introduce both classes of externalities into a model of capital accumulation, comparing their impacts on economic performance, and deriving the appropriate policy responses. Most of our attention is devoted to a stationary Ramsey model, but for comparative purposes an endogenous growth model of the Romer (1986) type is also briefly considered.[5] Our results take the form of a series of propositions summarizing how the externalities influence the macroeconomic equilibrium and proposing fiscal policies to correct for the distortions they may create.

One general conclusion is that consumption externalities will have long-run effects on production if and only if labor supply is endogenous. This is because such externalities affect the marginal valuation of consumption, which, if the leisure decision is endogenous, changes the optimal utility value of the marginal product of labor, thereby influencing long-run capital and output. If labor supply is fixed, then consumption externalities have no impact on either the steady-state capital stock or consumption. Moreover, for a range of commonly employed utility functions, including the isoelastic function, they will have no distortionary effect on the transitional path either. But, for other utility functions they do distort the transitional path, and we discuss an example that leads to over-consumption in the short run followed by subsequent under-consumption. In this case the distortion can be corrected by introducing a consumption tax (or subsidy) that is gradually reduced as the economy converges to equilibrium. With elastic labor supply, a negative consumption externality leads to long-run consumption, capital, and labor supply that are all too high relative to their respective optima, distortions that can be easily corrected by an appropriate consumption tax.

Production externalities always generate long-run distortions, irrespective of whether labor is

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[4] Two exceptions are Fisher and Hof (2000a, 2000b) who introduce a consumption externality ("conspicuous consumption") into a simple Ramsey model.

[5] That is, we abstract from population growth and accordingly, the Ramsey model is associated with a fixed long-run capital stock, while the endogenous growth model has long-run capital accumulation.

fixed or not. Thus a positive production externality leads to a sub-optimally low capital stock with under-consumption, a distortion that can be corrected by an appropriate subsidy to capital. The two types of externalities will interact if and only if labor supply is elastic. In that case, a consumption externality will affect the potency of the production externality on long-run consumption through its impact on the labor-leisure choice and hence on the marginal product of capital.

Analogous types of results are obtained for an endogenously growing economy of the Romer (1986) type, although the comparisons are in terms of equilibrium growth rates and consumption-capital ratios, rather than levels. There are, however, subtle but significant differences in the results obtained for the two types of models. For example, while with fixed labor supply, consumption externalities alone do not cause distortions, they nevertheless affect the sizes of the distortions caused by production externalities, an effect that does not exist in the Ramsey model.

The rest of the paper proceeds as follows. Section 2 first lays out the basic model, describing how the two types of externalities impinge on the economy, and then derives the macrodynamic equilibrium for both the decentralized and centrally planned economies. Section 3 compares the effects of the externalities on the steady-state equilibria, while Section 4 analyzes their dynamic effects in a simple example. Section 5 derives the tax policies that enable the decentralized economy to replicate the first-best optimum. Section 6 focuses on the special case where the production function is of the Romer (1986) form and the equilibrium is one of steady endogenous growth. Section 7 concludes, while some technical details are relegated to the Appendix.

2. Production and Consumption Externalities

2.1 Preferences and Technology

Consider an economy populated by *N* infinitely lived identical households, where *N* remains constant through time. Let *c* denote the consumption of the representative household, *L* the labor input supplied by the household, and *C* the average per capita consumption, $C = \sum c/N$. Following Abel (1990) and Gali (1994), we assume the utility function of the typical household depends not only on his own consumption, *c*, and leisure, $l \equiv 1 - L$, but also on the average per capita consumption level, *C*: $U(c, C, l) \equiv U(c, C, 1 - L)$. We denote the marginal utility of private consumption, per capita aggregate consumption, and leisure by U_c, U_C, and U_l, respectively. We assume that households derive positive marginal utility from their own consumption and leisure, i.e. $U_c > 0, U_l > 0$. In addition, the utility function is concave in these two quantities with $U_{cl} \geq 0$, so that the marginal utility of private consumption increases with leisure.

The key issue concerns the externality imposed by aggregate consumption on the well-being of the individual agent. In considering this, it is useful to contrast two distinct ways the externality may influence the individual's welfare. First, it may directly affect the agent's level of utility, given his own level of consumption. Thus, following Dupor and Liu (2003), we may say that the household feels either *jealous*, if $U_C < 0$, or *admiring*, if $U_C > 0$, when other agents' consumption increases. Second, the externality may influence the agent's marginal utility of his own consumption. This is often referred to as *keeping up with the Joneses*, for which different formulations can be found. Thus, e.g., Gali (1994), in a model abstracting from labor supply, describes it in terms of the cross partial derivative $U_{cC}(c,C)/U_c(c,C) > 0$. But this definition is not invariant with respect to the ordinal utility measure. For this reason, using a model with endogenous labor supply, Dupor and Liu (2003) specify it in terms of the effect of the average per capita consumption on the agent's marginal rate of substitution between his own consumption and leisure. That is, they define keeping up with the Joneses as, $d\left(U_c/U_l\right)/dC > 0$ and what they call "running away from the Joneses" as $d\left(U_c/U_l\right)/dC < 0$.

It is clear that the first type of externality is necessary for the existence of the second, but not vice versa.[6] Dupor and Liu (2003) documented that while jealousy and admiration are important for equilibrium over-consumption and under-consumption, keeping up with, or running away from, the Joneses are important for asset pricing. We show that with endogenous labor supply, jealousy and admiration are the crucial factors in determining the qualitative effects of consumption externalities on both the steady-state levels of capital (in the Ramsey model) and the equilibrium growth rates (in

[6] Thus if $U_C \equiv 0$ there is no externality to which the agent can respond. On the other hand, if the utility function is additively separable in aggregate consumption, any feelings of jealousy, for example, would have no impact on their individual behavior. However, most widely adopted utility functions are non-separable, in which case the two types of externalities are not independent. For example, the constant elasticity utility function $U(c,C) \equiv \left(1/(1-\gamma)\right)\left(cC^{\rho}\right)^{1-\gamma}$, implies the relationship: $\text{sgn}(U_C) = \text{sgn}(\rho)$; $\text{sgn}(U_{cC}) = \text{sgn}(\rho(1-\gamma))$. Thus if the intertemporal elasticity of substitution is less than 1 $(\gamma > 1)$, the agent will keep up with the Joneses [as defined by Gali (1994)], if and only if he is jealous.

the Romer model). Whether the preferences exhibit keeping up with, or running away from, the Joneses is reflected in the magnitudes of the long-run deviations from the optimum.

We shall impose the following restrictions on the consumption externality:

Assumption 1: (Consumption Externality) (i) $U_c + U_C > 0$

(ii) $U_{cc} + U_{cC} < 0$

(iii) $U_{lc} + U_{lC} \geq 0$

These conditions assert that either the externality augments the direct effect, or, if it is offsetting, it is dominated by the direct effect. Thus, for example, if the entire economy receives an additional unit of consumption, any negative effect on the utility of an individual agent, due to jealousy ($U_C < 0$), is dominated by the direct positive effect of his own increase in consumption.[7] These three conditions impose restrictions on the strength of these external consumption effects.

The household has a production technology that is homogeneous of degree one in its private inputs, capital k and labor L, with both factors having positive but diminishing marginal physical product. This latter property, together with the homogeneity, implies the restriction $F_{kL} > 0$. In addition, output depends on the average (or aggregate) stock of capital, denoted by $K = \sum k / N$. Thus the overall production function is $F(k,K,L)$ where $F_k(k,K,L)$ denotes the marginal product of private capital, and $F_K(k,K,L)$ the marginal product of aggregate capital.

Turning now to the production externality as specified by F_K, in the case that the average capital stock serves as a proxy for knowledge as in Romer (1986, 1989), we assume $F_K > 0$ so that the aggregate capital stock generates a positive production externality. However, $F_K < 0$ may arise in a circumstance in which production depends upon a publicly provided input that is subject to some degree of congestion; see e.g. Barro and Sala-i-Martin (1992), Eicher and Turnovsky (2000).[8]

Analogously, we impose the following restrictions on the production externality:

Assumption 2: (Production Externality) (i) $F_k + F_K > 0$

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[7] In equilibrium $U_c + U_C$ equals the social shadow value of capital and is presumably positive.

[8] Parallel characterizations of production externalities, to those suggested for consumption externalities, in terms of F_K and F_{Kk}, can be made. In particular, in equilibrium $F_k + F_K = \beta > 0$.

(ii) $F_{kk} + F_{kK} \le 0$

(iii) $F_{Lk} + F_{LK} \ge 0$

These conditions assert that either the externality reinforces the direct effect, or alternatively, if it is offsetting, e.g. if it is due to congestion, it is dominated by the direct effect. Thus, even if an increase in aggregate capital were to reduce the productivity of labor, when it occurs in conjunction with an increase in the individual's capital stock, labor productivity increases.

We should emphasize at the outset that we will focus on equilibrium paths along which all households are identical, so that $c = C,\ k = K$. We shall refer to such paths as "symmetric equilibria", and derivatives will be evaluated at these equilibrium points. Thus, e.g., $U_c(C,C,L) = \partial U(c,C,L)/\partial c\big|_{c=C}$ and likewise for the other derivatives.

2.2 Alternative Specifications of Consumption Externality

The formulation adopted for the consumption externality, while general with respect to the functional form, is nevertheless restrictive in that the utility function is time separable. While this formulation is reasonable and widely adopted [e.g. Boskin and Sheshinski (1978), Gali (1994), Harbaugh (1996), Ljungqvist and Uhlig (2000), Dupor and Liu (2003)], other specifications of this basic idea also exist. One class of such models specifies the reference consumption stock as being based on past average consumption in the economy, referring to this as "*catching* up with the Joneses". An optimal (efficient) allocation is then derived by treating the reference consumption stock as being generated by gradually evolving internal habits. This approach is adopted by Carroll, Overland, and Weil (1997, 2000) and Ljungqvist and Uhlig (2000), although none of these models consider production externalities.[9] Since in the long-run habits converge to a stationary level, the long-run implications of our model, insofar as the long-run distortions from the consumption externality and the appropriate corrective taxes are concerned, are not very different from what these models would imply. Rather, the main differences would be in the transitional dynamics, which

[9] Carroll, Overland, and Weil (1997, 2000) do not address efficiency issues, focusing entirely on the effects of these alternative utility functions in a simple AK growth model. Ljungqvist and Uhlig (2000) do address optimal tax issues, although they abstract from capital accumulation.

because of the gradually evolving reference consumption stock would need to be analyzed numerically.[10]

A few papers formulate the utility externality in terms of relative wealth, rather than consumption; see e.g. Kurz (1968) for an early reference and Zou (1995), Corneo and Jeanne (1997), Futagami and Shibata (1998) for more recent contributions. This specification is sometimes referred to as reflecting "status" and the "spirit of capitalism". Finally, de la Croix and Michel (1999) consider "inherited tastes" in which young agents acquire aspirations from their parents.[11]

2.3 Macrodynamic Equilibrium: Decentralized Economy

The representative household in the decentralized economy chooses consumption, labor supply, and its rate of capital accumulation to maximize the utility function

$$\int_0^\infty U(c, C, (1-L))e^{-\beta t}dt \tag{1a}$$

subject to the capital accumulation equation.

$$\dot{k} = F(k, K, L) - c \tag{1b}$$

In doing so, the household behaves atomistically, taking the aggregate quantities C and K as given.

Let starred variables denote the equilibrium of the decentralized economy. A symmetric equilibrium is then described by

$$U_c(C^*, C^*, 1-L^*) = \lambda^* \tag{2a}$$

$$U_l(C^*, C^*, 1-L^*) = \lambda^* F_L(K^*, K^*, L^*) \tag{2b}$$

$$F_k(K^*, K^*, L^*) = \beta - \frac{\dot{\lambda}^*}{\lambda^*} \tag{2c}$$

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[10]The specification of utility by time non-separable preferences dates back to Ryder and Heal (1973). Formally the specification of "rational addiction" due to Becker and Murphy (1988) closely resembles that of habit formation.

[11]Empirical evidence from developing countries suggests that nutrition improves productivity; see e.g. Leibenstein (1957). Proxying nutrition by aggregate consumption, this argument would motivate introducing aggregate consumption as an externality in the agent's production function.

$$\dot{K}^* = F(K^*, K^*, L^*) - C^* \tag{2d}$$

where λ^* is the *private* shadow value of an additional unit of capital. Equation (2a) equates the marginal utility of consumption to shadow value of capital. Equation (2b) equates the marginal utility of leisure to the marginal utility obtained from the additional output if labor input is increased by one more unit. Equation (2c) equates the rate of return on capital to the rate of return on consumption, while (2d) describes the rate of capital accumulation in equilibrium. From equations (2a) and (2b) we may solve for C^* and L^* in terms of K^* and λ^*. Substituting the resulting expressions into (2c) and (2d) we obtain an autonomous system describing the dynamics of the capital stock and its shadow value.

2.4 Macrodynamic Equilibrium: Centrally Planned Economy

In deriving his private optimum, the individual agent neglects the externalities present in both consumption and production. As a consequence the consumption and capital in a symmetric equilibrium may diverge from the socially optimal levels. To derive the optimal allocation of the economy, we consider a social planner who, in maximizing the objective (2) subject to the resource constraint $\dot{K} = F(K, K, L) - C$, takes both externalities into account.

Denoting the social optimum by tildes, the equilibrium conditions are now modified to

$$U_c(\tilde{C}, \tilde{C}, 1-\tilde{L}) + U_C(\tilde{C}, \tilde{C}, 1-\tilde{L}) = \tilde{\lambda} \tag{3a}$$

$$U_l(\tilde{C}, \tilde{C}, 1-\tilde{L}) = \tilde{\lambda} F_L(\tilde{K}, \tilde{K}, \tilde{L}) \tag{3b}$$

$$F_k(\tilde{K}, \tilde{K}, \tilde{L}) + F_K(\tilde{K}, \tilde{K}, \tilde{L}) = \beta - \frac{\dot{\tilde{\lambda}}}{\tilde{\lambda}} \tag{3c}$$

$$\dot{\tilde{K}} = F(\tilde{K}, \tilde{K}, \tilde{L}) - \tilde{C} \tag{3d}$$

In particular, $\tilde{\lambda}$ refers to the *social* shadow value of capital. The structure of (3) parallels that of (2) in that the first two equations can be solved for $\tilde{C}$ and $\tilde{L}$ in terms of $\tilde{K}$ and $\tilde{\lambda}$, which upon substitution yields an autonomous system in terms of $\tilde{K}$ and $\tilde{\lambda}$. Note, however, the key difference between (2a) and (3a) reflects the consumption externality, and the difference between (2c) and (3c) reflects the production externality. These in turn are reflected in the optimal equilibrium dynamics.

Finally, as our analysis will show, the significance of consumption externalities depends crucially upon whether labor supply is elastic or inelastic. In the latter case, the first order conditions for the labor/leisure decision [equations 2(b) and 3(b)] drop out, but the remaining equations are unchanged.

Before proceeding, an important technical issue should be addressed. The equilibrium conditions summarized in (2a) – (2d) and (3a) – (3d) are necessary for an interior optimum to obtain. As long as the utility function is concave and the production function has the usual neoclassical properties, and both have sufficient curvature, these equilibrium conditions suffice to ensure that the interior optimum both exists and is unique. But the presence of externalities may cause potential problems, in that the concavity conditions that ensure a unique interior optimum may no longer hold (see footnote 29). Throughout our analysis we shall simply maintain the assumption that the equilibrium conditions described by (2) and (3) always yield a unique interior optimum.

3. Comparison of Steady-State Equilibria

We begin by comparing the steady-state equilibrium in the simple case where labor is supplied inelastically.

3.1 Inelastic labor supply:

For the decentralized economy, the steady-state equilibrium satisfies

$$U_c(C^*,C^*) = \lambda^*, \tag{4a}$$

$$F_k(K^*,K^*) = \beta, \tag{4b}$$

$$F(K^*,K^*) = C^* \tag{4c}$$

In steady-state equilibrium, the marginal product of capital equals the time discount rate (equation 4b), which determines the equilibrium capital K^*. Because the representative household does not take account of the production externalities of capital, the equilibrium capital stock will generally be non-optimal. In the steady state, output is used solely for consumption. Once K^* is known, equation (4c) will determine the equilibrium consumption C^*.

The steady-state optimality conditions for the planner's problem are

$$U_c(\tilde{C},\tilde{C}) + U_C(\tilde{C},\tilde{C}) = \tilde{\lambda} \tag{5a}$$

$$F_k(\tilde{K},\tilde{K}) + F_K(\tilde{K},\tilde{K}) = \beta \tag{5b}$$

$$F(\tilde{K},\tilde{K}) = \tilde{C}. \tag{5c}$$

Comparing (5b) with (4b), we see that the equilibrium level of capital differs from its socially optimal level if and only if F_K is not zero. It then follows from (5c) and (4c) that deviations in the capital stock imply corresponding deviations in consumption. More precisely, we have

Proposition 1: *In an economy with inelastic labor supply, steady-state equilibrium capital and consumption are below their optimal levels ($K^* < \tilde{K},\ C^* < \tilde{C}$) if and only if there is a positive production externality ($F_K > 0$). They are above their optimal levels if and only if there is a negative production externality ($F_K < 0$). Consumption externalities have no effect on the steady-state equilibrium.*

Proof: Suppose $F_K > 0$. Then $F_k(K,K) < F_k(K,K) + F_K(K,K)$ for all K. Given $(F_{kk} + F_{kK}) < 0$, (4b) and (5b) imply $K^* < \tilde{K}$. Since $C = F(K,K)$, then $C^* < \tilde{C}$.

Suppose $C^* < \tilde{C}$ and $K^* < \tilde{K}$. Since $(F_{kk} + F_{kK}) < 0$, then $F_k(K^*,K^*) > F_k(\tilde{K},\tilde{K})$. Applying this result together with (4b) and (5b): $F_k(K^*,K^*) = \beta = F_k(\tilde{K},\tilde{K}) + F_K(\tilde{K},\tilde{K})$, we get $F_k(\tilde{K},\tilde{K}) < F_k(\tilde{K},\tilde{K}) + F_K(\tilde{K},\tilde{K})$. It therefore follows that $F_K > 0$.

Here, consumption externalities play no role in determining the steady state, either directly or through their interaction with production externalities. This is because the steady-state capital stock and therefore consumption is determined by the production technology alone. With exogenous labor supply, consumption externalities, which impact through the labor-consumption tradeoff, have no channel to affect steady-state output.

3.2 Elastic Labor Supply:

With endogenous labor supply, the steady-state symmetric equilibrium in the decentralized economy is characterized by the following conditions:

$$U_c(C^*,C^*,1-L^*)=\lambda^* \tag{6a}$$

$$U_l(C^*,C^*,1-L^*)=\lambda^* F_L(K^*,K^*,L^*) \tag{6b}$$

$$F_k(K^*,K^*,L^*)=\beta \tag{6c}$$

$$F(K^*,K^*,L^*)=C^* \tag{6d}$$

The corresponding equilibrium in the centrally planned economy is:

$$U_c(\tilde{C},\tilde{C},1-\tilde{L})+U_C(\tilde{C},\tilde{C},1-\tilde{L})=\tilde{\lambda} \tag{7a}$$

$$U_l(\tilde{C},\tilde{C},1-\tilde{L})=\tilde{\lambda}F_L(\tilde{K},\tilde{K},\tilde{L}) \tag{7b}$$

$$F_k(\tilde{K},\tilde{K},\tilde{L})+F_K(\tilde{K},\tilde{K},\tilde{L})=\beta \tag{7c}$$

$$F(\tilde{K},\tilde{K},\tilde{L})=\tilde{C}. \tag{7d}$$

Because of the endogenous labor decision, consumption externalities will now also affect the steady state. This is because they affect the marginal valuation of consumption, which in turn changes the optimal utility value of the marginal product of labor. Therefore, consumption distortion results in labor distortion, and thus creates production inefficiency. This effect operates even if the utility function is additively separable in consumption and leisure; see (A.2)

To compare the steady-state equilibrium with the optimal steady-state allocation, we apply Taylor expansions to equations (6a)-(6d) around the optimal steady state ($\tilde{C},\tilde{K},\tilde{L},\tilde{\lambda}$) defined in (7a)-(7d). This leads to the following proposition relating the actual and socially optimal equilibria in the presence of consumption and production externalities. The proof is provided in the Appendix.

Proposition 2: *In an economy with endogenous labor supply, the steady-state* *equilibrium has the following properties.*

1. *In the case of a negative consumption externality (i.e. preferences exhibit jealousy* $(U_C < 0)$*), equilibrium consumption, capital stock, and labor supply are all greater than their respective long-run optimal values,* $(C^* > \tilde{C},\ K^* > \tilde{K},\ L^* > \tilde{L})$*. In the case of admiration these relationships are reversed.*

2. *If the production technology exhibits a positive aggregate capital externality* $(F_K > 0)$*, equilibrium consumption and capital stock are below their respective long-run optimal values,* $(C^* < \tilde{C},\ K^* < \tilde{K})$*. However, the long-run labor supply may be either greater than or less than its long-run optimal value* $L^* \gtrless \tilde{L}$*. In the case of a negative externality these relationships are reversed.*

To understand the result, we first consider the effect of a consumption externality. The first order condition in the household's maximization problem requires the marginal rate of substitution between consumption and leisure to equal the marginal product of labor. In the case of jealousy, equilibrium consumption is over-valued. This results in the equilibrium consumption being too high compared to the social optimum and leisure being too low, (or equivalently, labor supply being too high). Because capital and labor are complements $F_{kL} \geq 0$ and because in steady state the marginal product of capital is equal to β, a higher equilibrium labor input implies a higher equilibrium capital stock. The argument is reversed if preferences exhibit admiration.[12] From the expressions for the deviations of the decentralized equilibrium from the optimum provided in the Appendix, (equations A.2), we see that the consumption externality due to keeping up with the Joneses affect the magnitudes of the long-run deviations.

Next consider the effect of a positive production externality. In this case the marginal product of capital is undervalued by the private agent. But since the marginal product of capital is diminishing, and the steady state requires the marginal product of capital to equal β, less capital is needed in the decentralized economy compared to the socially optimal economy to achieve the

[12] In his book *Luxury Fever* Robert Frank (1999) describes the over-consumption phenomenon and argues that people tend to work too long, due to negative consumption externalities, consistent with the result of Proposition 2. However, contrary to what he argues, Proposition 2 shows that negative consumption externalities do not imply under-saving. In fact, with negative consumption externalities, the level of steady-state capital is too high, not too low. This result is consistent with Proposition 1c of Fisher and Hof (2000b).

equilibrium condition on the marginal product of capital. Thus less output will be produced, thereby reducing equilibrium consumption. The production externality has two offsetting effects on labor. First, lower consumption means a higher marginal utility of consumption. This requires a higher equilibrium labor supply in order for the condition that the marginal rate of substitution between consumption and leisure equals marginal product of labor to hold. On the other hand, because capital and labor are complements, a lower capital stock will decrease the marginal product of labor and thus decrease the labor input. The net effect on equilibrium labor supply is therefore ambiguous.

4. Dynamics

Externalities not only create a divergence between the long-run competitive equilibrium and the socially optimal outcome, but they also affect the transitional dynamics. In this section we illustrate this by comparing the transition paths followed by the competitive economy and the centrally planned economy in response to an exogenous increase in the time discount rate β.

We will restrict our discussion of dynamics to an economy with exogenous labor supply. In this case, (2a) – (2d) implies that the dynamics of the decentralized equilibrium can be described by:

$$\frac{\dot{C}^*}{C^*} = \theta_d(C^*)\left(F_k(K^*, K^*) - \beta\right) \tag{8a}$$

$$\dot{K}^* = F(K^*, K^*) - C^* \tag{8b}$$

where for convenience the fixed labor supply is omitted and

$$\theta_d \equiv -\frac{U_c(C^*, C^*)}{C^*\left(U_{cc}(C^*, C^*) + U_{cC}(C^*, C^*)\right)} > 0 \tag{8c}$$

represents the effective intertemporal elasticity of substitution in the decentralized economy. Equation (8a) is obtained by taking the time derivative of (2a) and combining it with (2c). The steady-states to (8a) and (8b) are described by (4b) and (4c) respectively.

Following the same steps for the centrally planned economy leads to

$$\frac{\dot{\tilde{C}}}{\tilde{C}} = \theta_p(\tilde{C})\left(F_k(\tilde{K}, \tilde{K}) + F_K(\tilde{K}, \tilde{K}) - \beta\right) \tag{9a}$$

$$\dot{\tilde{K}} = F(\tilde{K}, \tilde{K}) - \tilde{C} \tag{9b}$$

where
$$\theta_p \equiv -\frac{U_c(\tilde{C},\tilde{C}) + U_C(\tilde{C},\tilde{C})}{\tilde{C}\left(U_{cc}(\tilde{C},\tilde{C}) + 2U_{cC}(\tilde{C},\tilde{C}) + U_{CC}(\tilde{C},\tilde{C})\right)} > 0 \tag{9c}$$

is the effective intertemporal elasticity of substitution in the centrally planned economy.[13] The steady-states to (9a), (9b) are described by (5b) and (5c), respectively.[14]

Comparing these two sets of equations brings out an interesting contrast between how the production externality and the consumption externality influence the transitional path. The former does so by shifting the steady-state capital stock, whereas the latter operates through the effective intertemporal elasticity of substitution. For simplicity, we assume that there is a consumption externality but no production externality. In this case the dynamics in the two economies simplify to

$$\frac{\dot{C}^*}{C^*} = \theta_d(C^*)\left(F_k(K^*) - \beta\right) \tag{8a'}$$

$$\dot{K}^* = F(K^*) - C^* \tag{8b'}$$

and

$$\frac{\dot{\tilde{C}}}{\tilde{C}} = \theta_p(\tilde{C})\left(F_k(\tilde{K}) - \beta\right) \tag{9a'}$$

$$\dot{\tilde{K}} = F(\tilde{K}) - \tilde{C} \tag{9b'}$$

With the common steady state (c.f. Proposition 1), denoted by a bar, we obtain

$$F_k(\bar{K}^*) = F_k(\bar{\tilde{K}}) = \beta \tag{10a}$$

$$F(\bar{K}^*) - \bar{C}^* = F(\bar{\tilde{K}}) - \bar{\tilde{C}} = 0 \tag{10b}$$

From equations (10a) and (10b), we see that an increase in β lowers the steady-state levels of capital

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[13]In describing both (8c) and (9c) we adopt the terminology of Fisher and Hof (2000a). These measures are modifications of the usual definitions of the intertemporal elasticity of substitution to take account of the respective impacts of the consumption externality in the two economies.

[14]The fact that θ_p and θ_d are positive is ensured by the restrictions we have imposed on the utility function and in particular on the magnitude of the consumption externality.

and consumption by the common amounts

$$\frac{d\bar{K}^*}{d\beta} = \frac{d\bar{\tilde{K}}}{d\beta} = \frac{1}{F_{kk}} < 0 \tag{11a}$$

$$\frac{d\bar{C}^*}{d\beta} = \frac{d\bar{\tilde{C}}}{d\beta} = \frac{F_k}{F_{kk}} < 0 \tag{11b}$$

a convenient property that facilitates the comparison of the transitional paths of the two economies.

With the two economies having the same steady states, the consumption externalities cause the transitional dynamics of the decentralized equilibrium to deviate from that of the socially optimal path to the extent that θ_d deviates from θ_p. Thus, comparing (8c) and (9c), we see that consumption externalities will *not* cause any distortions if and only if:

$$\frac{U_c(C,C)}{C\left(U_{cc}(C,C)+U_{cC}(C,C)\right)} = \frac{U_c(C,C)+U_C(C,C)}{C\left(U_{cc}(C,C)+2U_{cC}(C,C)+U_{CC}(C,C)\right)} \tag{12}$$

It can be verified that equation (12) is equivalent to the following condition:

$$\left.\frac{\partial}{\partial c}\left(\frac{U_C}{U_c}\right)\right|_{c=C} + \left.\frac{\partial}{\partial C}\left(\frac{U_C}{U_c}\right)\right|_{c=C} = 0 \tag{13}$$

Condition (13) is equivalent to (13'), namely that the equilibrium marginal rate of substitution between private and public consumption be constant through time, i.e.[15]

$$\left.\left(\frac{U_C}{U_c}\right)\right|_{c=C} \text{ is constant}. \tag{13'}$$

We summarize the result for allocation in the following proposition.[16]

Proposition 3: *In an economy with inelastic labor supply, consumption externalities*

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[15] This can be seen immediately as follows. Let $G(C,C) \equiv \left(U_C/U_c\right)\big|_{c=C}$ Taking the time derivative of $G(C,C)$ and using (13) we see that $\dot{G}(C,C) \equiv 0$, i.e. $G(C,C) \equiv \left(U_C/U_c\right)\big|_{c=C}$ is constant, as asserted.

[16] A similar result has been obtained by Fisher and Hof (2000a). They also obtain what they term an "observational equivalence" result. This states that consumption externalities have *no* influence on the dynamic path if the effective decentralized intertemporal elasticity of substitution equals the intertemporal elasticity of substitution in an economy without consumption externalities.

have no <u>distortionary</u> effect on the resource allocation along the transitional dynamic path if and only if the marginal rate of substitution between private and public consumption is constant through time.

Condition (13) asserts that in equilibrium, the overall effect of a uniform increase in private consumption plus aggregate consumption on the agent's marginal rate of substitution between these two quantities is zero. In other words, *increases* in *private* and *aggregate* consumption of equal magntiude have exactly offsetting effects on the marginal rate of substitution, and (13') indicates that this will be so if and only if the marginal rate of substitution is constant.

As it turns out, many standard utility functions satisfy the condition in Proposition 3. For example, the CES utility function

$$U(c,C)=\frac{1}{1-\gamma}\left[\left(\frac{c^{\varphi}-\kappa C^{\varphi}}{1-\kappa}\right)^{\frac{1}{\varphi}}\right]^{1-\gamma} \tag{14a}$$

employed by Dupor and Liu (2003), which nests several existing utility specifications used in the literature, satisfies condition (13).[17] So do the constant elasticity utility function, which has been widely employed in endogenous growth models:[18]

$$U(c,C)=\frac{1}{1-\gamma}\left(cC^{-\kappa}\right)^{1-\gamma}, \tag{14b}$$

and the following multiplicatively and additively separable utility functions:

$$U(c,C)=AU(c)\left[U(C)\right]^{\kappa} \tag{14c}$$

$$U(c,C)=A+U(c)+\kappa U(C) \tag{14d}$$

In any of these cases, jealousy or admiration does affect the household's utility level, but it does not create distortions relative to the socially optimal consumption and capital accumulation

[17]This function nests the specifications (with minor parameter adjustments) employed by Gali (1994) and Ljungqvist and Uhlig (2000).
[18]Most endogenous growth models do not include consumption externalities. Carroll, Overland, and Weil (1997) adopt this specification, although they assume that the reference consumption level evolves slowly over time.

decisions. Thus, for example, the simple constant elasticity utility function (14b) implies

$$\theta_d = \theta_p = \frac{1}{1-(1-\gamma)(1-\kappa)}$$

Assuming that the intertemporal elasticity of substitution is less than 1, so that $\gamma > 1$, we see that a negative consumption externality (i.e. jealousy $\kappa > 0$) increases the instantaneous speed of adjustment of consumption (by identical amounts) in both economies.[19]

But condition (13) can be violated. As a simple example consider the familiar constant elasticity utility function where the negative consumption externality appears additively

$$U(c,C) = \frac{1}{\chi_1}c^{\chi_1} + \frac{1}{\chi_2}AC^{\chi_2} \qquad A<0,\ \chi_1<1,\ \chi_2<1 \tag{14e}$$

In this case we see:

$$\theta_d = \frac{1}{1-\chi_1};\quad \theta_p = \theta_d\left[1+\frac{A(\chi_2-\chi_1)}{(1-\chi_1)C^{\chi_1-\chi_2}+A(1-\chi_2)}\right]$$

so that θ_d deviates from θ_p as long as $\chi_1 \neq \chi_2$.

Comparing (8a') and (8b') with (9a') and (9b'), respectively, we see that the dynamics of the two economies have the same structure, differing only with respect to the value of the effective intertemporal elasticity of substitution. We therefore represent these equations by the general forms:

$$\frac{\dot{C}}{C} = \theta\left(F_k(K)-\beta\right) \tag{15a}$$

$$\dot{K} = F(K) - C \tag{15b}$$

Taking a linear approximation around the steady state yields

$$\begin{bmatrix}\dot{K}\\ \dot{C}\end{bmatrix} = \begin{bmatrix}F_k & -1\\ \theta F_{kk}\bar{C} & 0\end{bmatrix}\begin{bmatrix}K-\bar{K}\\ C-\bar{C}\end{bmatrix} \tag{16}$$

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[19]In the case of (14a) and (14d) the consumption externality has no effect on $\theta_d (= \theta_p)$. In contrast, (14c) affects both equally, in an analogous fashion to the effect of (14b).

where $\bar{K}, \bar{C}$ denote the (common) steady-state values. The dynamics are thus a saddlepoint, with eigenvalues $\mu_1 < 0$ and $\mu_2 > 0$. We will focus on the stable path, the evolution of which is given by

$$K(t) = \bar{K} + \left(K_0 - \bar{K}\right)e^{\mu_1 t} \tag{17a}$$

$$C(t) - \bar{C} = \left(F_k - \mu_1\right)\left(K(t) - \bar{K}\right) \tag{17b}$$

To compare the transition to the steady state, we consider the case where the consumption externality is such that $\theta_d > \theta_p$, which, for example, corresponds to $\chi_2 > \chi_1$ in the utility function (14e). The analysis of the case where the externality is such that $\theta_d < \theta_p$ is analogous. From the characteristic equation of (16), $\mu^2 - F_k\mu + \theta F_{kk}\bar{C} = 0$, we can easily show that $d\mu_1/d\theta < 0$, implying that an increase in θ makes μ_1 more negative, and thus speeds up the rate of adjustment.

Let μ_d and μ_p be the negative eigenvalues for the decentralized and the social planner's economy, respectively. Since $\theta_d > \theta_p$, it follows that $\mu_d < \mu_p < 0$, so the decentralized economy converges faster to the steady state than does the socially planned economy. Indeed, for a given initial capital stock, the two economies have different paths of convergence. Applying (17a) to the two economies we see that

$$K^*(t) - \tilde{K}(t) = \left(K_0 - \bar{K}\right)\left(e^{\mu_d t} - e^{\mu_p t}\right) < 0 \tag{18a}$$

so that the capital stock in the decentralized economy is always less than the socially optimal amount during the transition as illustrated in Fig. 1.A.

Recalling (17b), we see that the coefficient on the right hand side is positive, which means that both economies have a positively sloped stable saddle path in consumption-capital space. And since $\mu_d < \mu_p < 0$, the slope of the stable path is steeper for the decentralized economy, as illustrated in Fig. 1B. Starting from the initial equilibrium A, the decentralized economy initially jumps to C, and then proceeds along CD thereafter; the centrally planned economy follows the locus ABD.[20]

Applying (17) to both economies and taking the difference of the resulting equations, we get

$$C^*(t) - \tilde{C}(t) = (K_0 - \bar{K})\left[\left(F_k - \mu_d\right)e^{\mu_d t} - \left(F_k - \mu_p\right)e^{\mu_p t}\right] \tag{18b}$$

[20]The initial equilibrium corresponds to the lower rate of time preference, β.

Assume that the initial capital stock is higher than the steady-state capital, $K_0^* = \tilde{K}_0 \equiv K_0 > \overline{K}$. Equation (18b) then implies that initial consumption in the decentralized economy will exceed the socially optimal level: i.e. $C^*(0) = \tilde{C}(0) + (K_0 - \tilde{K})(\mu_p - \mu_d) > \tilde{C}(0)$. But since $\mu_d < \mu_p < 0$, it also follows that equilibrium consumption in the decentralized economy will decrease and converge to the steady state at a faster rate. We can calculate the crossover point of the consumption paths of the two economies by setting equation (18b) to zero and solve for *t*:

$$T = \frac{1}{\mu_p - \mu_d} \ln\left(\frac{F_k - \mu_d}{F_k - \mu_p}\right) \tag{19}$$

Fig. 1.C plots the transitional time paths for consumption in the two economies. Prior to time 0 the economy is following the path PP, with both the decentralized and centrally planned economies enjoying the same level of consumption. At time 0, when the increase in the rate of time preference occurs, consumption in the decentralized economy jumps to Q above R, in the centrally planned economy. The agent in the decentralized economy overvalues consumption and therefore initially over-consumes, relative to what is socially optimal. At the same time, he also reduces his consumption at a faster rate, so that prior to time *T*, defined by (19), over-consumption occurs in the decentralized economy compared to the social planner's economy, whereas under-consumption will occur along the remainder of the transitional path, thereafter. This is a consequence of the under-investment that is occurring in the decentralized economy, as illustrated in Figure 1.A.

With equilibrium consumption exceeding optimal consumption prior to time T, during the early phase of the transition the welfare in the decentralized economy exceeds that associated with the intertemporally optimal path, although these are offset by the relative losses after time T, when under-consumption occurs. The difference in intertemporal welfare is obtained by calculating

$$\int_0^\infty \left[U(C^*, C^*) - U(\tilde{C}, \tilde{C})\right] e^{-\beta t} dt \tag{20}$$

An approximation to this quantity can be computed by linearizing $U(C^*, C^*) - U(\tilde{C}, \tilde{C})$ about the steady-state level, substituting (18b), and evaluating the integral (20). Performing this calculation while noting (18a) and (18b), yields

$$U(C^*,C^*)-U(\tilde{C},\tilde{C})=\left(U_c(\bar{C},\bar{C})+U_C(\bar{C},\bar{C})\right)\left(C^*(t)-\tilde{C}(t)\right)$$

$$=\left(U_c+U_C\right)(K_0-\bar{K})\left[\left(\beta-\mu_d\right)e^{\mu_d t}-\left(\beta-\mu_p\right)e^{\mu_p t}\right] \qquad (21)$$

Finally, substituting (21) into (20) and evaluating we find that the welfare change is zero. That is, to a first order approximation, the present value of the welfare gains from over-consumption in the short run just offset the relative losses suffered after time *T*. In other words, the welfare loss resulting from not following the optimal path is of the second order. This is not too surprising, since the two economies have the same initial and final steady states, so that they deviate only during the transition.[21] Our results may be summarized as follows:

Proposition 4: *If labor supply is inelastic, consumption externalities do not distort the steady state. However, consumption externalities will distort the transitional path to the steady state if the marginal rate of substitution between private and public consumption is not constant, that is for any utility function that does not satisfy (13). If the initial level of the capital stock is above (below) the steady state, initial over (under)-consumption is followed by subsequent under (over)-consumption. To a first order approximation, the welfare gains during the first phase just match the subsequent losses during the second phase*.

The dynamics in the case of a production externality can be analyzed similarly, though the details are more complicated due to the fact that the initial and final steady states are affected by the externality. The fact that this externality involves permanent steady-state effects that differ between the decentralized and centrally planned economies (in contrast to only transitory effects along the transitional paths) means that the equilibrium path is associated with first-order welfare losses.

5. Optimal Tax Policy

The fact that consumption and production externalities create distortions in resource

[21]To compute the second-order approximation to the welfare change is more difficult, requiring a second-order approximation to the dynamics (15a), (15b) and then a second-order approximation to the welfare integral.

allocation provides the opportunity for government tax policies to improve efficiency. This section characterizes a tax structure that enables the decentralized equilibrium to replicate the first-best optimum of the centrally planned economy in the presence of such externalities.

Consider again the decentralized economy populated by identical households. Let τ_k be the tax rate on the return to private capital, τ_w the tax rate on labor income, τ_c the tax rate on consumption, and υ lump-sum transfers (taxes). The representative household maximizes the utility function (1a), subject to the budget constraint, now expressed as

$$\dot{k} = (1-\tau_k)rk + (1-\tau_w)wL - (1+\tau_c)c + \upsilon \tag{22}$$

where $r = F_k$ is the return to capital, $w = F_L$ is the wage rate. The government maintains a balanced budget, returning all the tax revenues to the households in the form of lump sum transfers:

$$\tau_k rK + \tau_w wL + \tau_c C = \upsilon \tag{23}$$

5.1 Inelastic Labor Supply

We begin with the case of inelastic labor supply. Suppressing the fixed labor supply, macroeconomic equilibrium comprises the first order conditions for the household's maximization,

$$U_c(C^*, C^*) = (1+\tau_c)\lambda^* \tag{24a}$$

$$(1-\tau_k)F_k(K^*, K^*) = \beta - \frac{\dot{\lambda}^*}{\lambda^*} \tag{24b}$$

together with the goods market equilibrium condition, (2d). In writing (24a) and (24b) we have imposed the symmetric equilibrium conditions $c = C$ and $k = K$ together with $r = F_k$.

Our objective is to determine a tax structure such that the decentralized economy replicates the dynamic equilibrium time path of the centrally planned economy, as described by (3a), (3c) and (3d), [with the fixed labor supply correspondingly suppressed]. To achieve this, we allow the tax rates, τ_k, τ_c to be time varying[22]. Replication involves setting these tax rates such that $K^* = \tilde{K}, C^* = \tilde{C}$, which then requires $\dot{\lambda}^*/\lambda^* = \dot{\tilde{\lambda}}/\tilde{\lambda}$, or equivalently $a\lambda^* = \tilde{\lambda}$, where a is an arbitrary

[22] With fixed labor supply, the tax on wage income τ_w is like a lump-sum tax and has no impact on the equilibrium.

constant.

Setting the tax rates in accordance with

$$\tau_k(t) = -\frac{F_K(K^*, K^*)}{F_k(K^*, K^*)} \tag{25a}$$

$$1 + \tau_c(t) = a\frac{U_c(C^*, C^*)}{U_c(C^*, C^*) + U_C(C^*, C^*)} \tag{25b}$$

ensures that $K^* = \tilde{K}, C^* = \tilde{C}$ during the entire transition (including time 0). Both tax rates are time-varying and will converge to constant levels as capital and consumption converge to their respective steady-state equilibrium values. The first equation asserts that the optimal tax on capital should be negative or positive according to whether the production externality is positive or negative.[23]

Equation (25b) implies that the *level* of the consumption tax is arbitrary. This is because with fixed labor supply the distortion resulting from the consumption externality vanishes in steady state, so that in long-run equilibrium the consumption tax acts like a lump-sum tax. Its steady-state value can therefore be set arbitrarily, being offset by the lump-sum rebate υ in (23). Instead, what is important is how the consumption tax is adjusted over time to correct for the distortions along the transitional path.[24]

There are several natural restrictions one can impose to determine a. The first is to set $a = 1$, so that $\lambda^* = \tilde{\lambda}$. In this case, (25b) reduces to[25]

$$\tau_c(t) = -\frac{U_C(C^*, C^*)}{U_c(C^*, C^*) + U_C(C^*, C^*)} \tag{25b'}$$

so that the tax (subsidy) on consumption is proportional to the size of the externality, relative to the

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[23] There is a presumption in much of the optimal tax literature that the tax on capital income should be low or zero; see Chamley (1986), Judd (1985). The present result that the optimal tax on capital should be negative in the presence of a positive production externality goes beyond that proposition. Note, however, that Judd (2002) has also shown that in an economy with imperfect competition the optimal tax on capital should be negative. Essentially, the externality in the present analysis is playing the role of the market imperfection in Judd's analysis.

[24] Letting $\pi(t) \equiv U_C(C^*, C^*) / \left(U_c(C^*, C^*) + U_C(C^*, C^*)\right)$ denote the relative size of the consumption externality to the total marginal utility of consumption, this can be seen explicitly from the time derivative of (25b) $\left(\dot{\tau}_c(t)/(1+\tau_c(t))\right) = -\left(\dot{\pi}(t)/(1-\pi(t))\right)$

[25] The optimal consumption tax (subsidy) reduces to that derived by Ljungqvist and Uhlig (2000, Proposition 1) for the specific utility function employed in their analysis of short-run demand management.

total marginal benefits of consumption. Alternatively, one can argue that since with fixed labor supply the distortion from the consumption externality vanishes in steady state, there is no need to tax steady-state consumption and we may impose the condition $\lim_{t\to\infty}\tau_c(t)=0$. Invoking this condition, (25b) yields the optimal consumption tax[26]

$$\tau_c(t)=\frac{\overline{(U_C/U_c)}-(U_C/U_c)}{1+(U_C/U_c)} \tag{25b''}$$

where the bar denotes the steady-state value of the marginal rate of substitution between the household's own and the economy-wide average consumption. In this case, the optimal consumption tax depends upon the difference between the steady-state and the current marginal rate of substitution between private and the economy-wide consumption. It converges to zero, as the economy converges to its steady state.

For the important class of utility functions for which (13') holds, (U_C/U_c) is constant and is at its steady-state level at all times. There is no consumption distortion, and the consumption tax should be constant (zero) at all times.

We may summarize these results as follows:

Proposition 5: *In an economy with exogenous labor supply, the entire time path of the optimal resource allocation can be obtained by setting taxes at each instant of time in accordance with*

$$\tau_k=-\frac{F_K}{F_k}\,;\;1+\tau_c(t)=a\frac{U_c(C^*,C^*)}{U_c(C^*,C^*)+U_C(C^*,C^*)},$$

where a is an arbitrary constant. As the economy approaches its steady state, these taxes converge to their steady-state rates. Furthermore, if condition (13) holds, then only the capital tax is needed to achieve the optimal transition path. With exogenous labor supply the tax on wage income can be set arbitrarily.

5.2 Elastic Labor Supply

[26]Considering (25b) in steady state and setting $\tau_c=0$, implies $a=1+\overline{(U_C/U_c)}$, which upon substitution into (25b) then yields (25b'').

When labor supply is endogenous, the equilibrium of the decentralized economy is now determined by the following conditions:

$$U_c(C^*, C^*, 1-L^*) = (1+\tau_c)\lambda^* \tag{26a}$$

$$U_l(C^*, C^*, 1-L^*) = (1-\tau_w)\lambda^* F_L(K^*, K^*, L^*) \tag{26b}$$

$$(1-\tau_k)F_k(K^*, K^*, L^*) = \beta - \frac{\dot{\lambda}^*}{\lambda^*} \tag{26c}$$

$$\dot{K}^* = F(K^*, K^*, L^*) - C^* \tag{26d}$$

which now needs to be compared to the corresponding relationships for the centrally planned economy, (3a) – (3d). Replication again requires that $a\lambda^* = \tilde{\lambda}$, in which case comparing (26b) to (3b) we see that this is consistent with an arbitrary constant tax on labor income, $\overline{\tau}_w$, identifying $a = (1-\overline{\tau}_w)$. Given the tax on labor income, the consumption tax

$$\frac{1+\tau_c(t)}{1-\overline{\tau}_w} = \frac{U_c(C^*, C^*, 1-L^*)}{U_c(C^*, C^*, 1-L^*) + U_C(C^*, C^*, 1-L^*)} \tag{27}$$

together with the capital income tax rate (25a) will ensure the replication of the first-best equilibrium. Thus we establish the following optimal taxation result.

Proposition 6: *In an economy with endogenous labor supply, the entire time path of the optimal resource allocation can be obtained by setting taxes at each instant of time in accordance with*

$$\tau_k(t) = \frac{-F_K(K^*, K^*, L^*)}{F_k(K^*, K^*, L^*)},\ \tau_w = \overline{\tau}_w,\ \text{and}\ \ \frac{1+\tau_c(t)}{1-\overline{\tau}_w} = \frac{U_c(C^*, C^*, L^*)}{U_c(C^*, C^*, L^*) + U_C(C^*, C^*, L^*)}.$$

The consumption and capital income taxes are time-varying and converge to constants as the economy converges to its steady state. Again, the tax on capital corrects the resource distortion from the production externality while the consumption tax corrects the distortion from the consumption externality. Although both externalities generate distortions in labor supply, no active labor income tax is needed once the source of distortions is rectified. The labor income tax can thus be set to zero.

The government should subsidize capital investment if there is a positive capital externality, and should levy a positive capital income tax if there is a negative capital externality. Similarly, private consumption should be subsidized if there is a positive consumption externality and should be subject to taxes if there is a negative consumption externality.[27]

The sizes of the required corrected taxes are likely to be quite significant, even for externalities of only a modest size. To give some idea of their numerical magnitudes, suppose the production function F is of the following Cobb-Douglas form $Ak^{\alpha}L^{1-\alpha}K^{\beta}$, while utility is given by (14b). In that case, $\tau_k = -\beta/\alpha$, while setting $\tau_w = 0$, implies $\tau_c = \kappa/(1-\kappa)$. Assuming a capital elasticity of $\alpha = 0.35$ and an externality of $\beta = 0.10$, both highly plausible values, implies an optimal subsidy to capital of almost 29%. Also, assuming a modest negative consumption externality, $\kappa = 0.20$, implies an optimal consumption tax of 25%.

The introduction of endogenous labor supply introduces three differences worth emphasizing. First, with inelastic labor supply, the optimal consumption tax includes a degree of arbitrariness (c.f. the presence of the arbitrary constant *a*). But when labor is supplied elastically, this arbitrariness disappears, once the fixed wage income tax is set. This is because λ becomes important for the labor-leisure choice and needs to be mimicked to attain the first-best optimum. Second, the arbitrariness of the tax on labor income with inelastic labor means that the first-best optimum in that case could in fact be reached with a uniform income tax, in which the income from capital and (fixed) labor are taxed at the same time-varying rate. By contrast, since with elastic labor supply the tax on labor income must remain fixed, these two components of income must necessarily be taxed at different rates. This is because the capital income tax, which remains unchanged from the case of inelastic labor supply, is necessary to correct for the distortion to the capital market, while the labor market is distortion-free. Finally, because the consumption externality distorts the steady-state leisure-consumption tradeoff, as long as the tax on labor income is set arbitrarily, the consumption tax is no longer in general transitory, since it must correct the steady-state distortion. The one exception is if $\bar{\tau}_w = -\overline{(U_C/U_c)}$ (where the bar denotes steady state), in which case τ_c

[27] Other tax structures may also attain the first-best optimum. Basically there are two distortions, one involving consumption, the other production, and therefore two tax rates -- one for each distortion -- are required for their correction. Thus, for example, the tax on capital income could be replaced by an appropriately determined investment subsidy, in conjunction with the consumption tax rate specified in Proposition 6.

reduces to (25b") and will again be transitory. In that case, the tax on labor income alone fully corrects the steady-state leisure-consumption distortion.

The presence of capital accumulation also influences the appropriate tax policy to correct for the consumption externality. Employing a static model in which labor is the only factor of production, Dupor and Liu (2003) showed that the consumption externality can be corrected by using a general income tax, which in their study is a tax on labor. But with capital accumulation, a general income tax would introduce a distortion into the capital market. Moreover, since there is no distortion in the labor market, the appropriate policy is to correct the consumption distortion directly, by taxing or subsidizing consumption, rather than labor income.

The time-varying optimal tax structures summarized in Propositions 5 and 6 yield interesting contrasts with optimal tax policies in other areas of public finance. For example, the well known tax smoothing result of Barro (1979) in which the government's objective is to finance some exogenous stream of expenditures at minimum cost, implies that under plausible conditions the optimal tax rate should be constant over time. It also contrasts with the Judd (1985) - Chamley (1986) result asserting that the steady-state tax on capital income should be zero. Here, the tax on capital is a Pigovian tax, directed at correcting a time-varying productive distortion. In the absence of such a distortion capital income should also remain untaxed.

6. Endogenous Growth

Externalities play an important role in the literature on endogenous growth models. Indeed, positive production externalities, generated by the presence of aggregate capital, provided the cornerstone of Romer's (1986) seminal contribution. This section extends the previous classification of production and consumption externalities to a Romer-type endogenous growth, studying their effects on the equilibrium and deriving the tax structure that will eliminate the distortions.

To generate an equilibrium of ongoing growth, the utility and production functions must yield a constant equilibrium consumption-capital ratio, with capital growing at a constant rate. This requires that the utility function be of the constant elasticity form [as in (28) and (35) below]. On the production side, ongoing growth is ensured for *any* production function that is homogeneous of degree one in private and economy-wide capital stock.[28] For expositional convenience we shall employ the standard Cobb-Douglas production function that dominates this literature, but the analysis extends without difficulty.

6.1 Inelastic Labor Supply

Consider the economy with exogenous labor supply. The preferences of the representative household are now represented by the following constant elasticity utility function:

$$\int_0^\infty \frac{1}{1-\gamma}\left(cC^\rho\right)^{1-\gamma} e^{-\beta t}dt \tag{28}$$

where $\gamma > 0$. The preference function (28) thus exhibits jealousy if $\rho < 0$, and admiration if $\rho > 0$. In addition, to ensure that conditions (i) and (ii) of Assumption 1 are met, we require $\rho > -1$ and $1-(1+\rho)(1-\gamma) > 0$. Before proceeding, we recall that the utility function satisfies the symmetry condition of Proposition 3.[29] Thus, although the presence of the consumption externality does affect the equilibrium, it does not alone introduce inefficiency relative to the optimum.

The production function follows the standard form:[30]

$$\dot{k} = Ak^\alpha K^{1-\alpha} - c \qquad A > 0,\ 0 < \alpha \le 1 \tag{29}$$

so that the household's production technology has positive externalities from aggregate capital, the magnitude of which is given by $(1-\alpha)$. Thus, although the individual's production function has diminishing returns to scale in private capital, the externality ensures that the equilibrium aggregate production function has constant returns to scale in the aggregate capital stock.

It is well known that for this specification of preferences and technology, the initial consumption level jumps to ensure that the economy is always on its balanced growth path, with

[28] See Turnovsky (2003) where a general stochastic production function of this form is developed.

[29] The utility function (28) was initially introduced by Abel (1990) and has more recently been employed by Carroll, Overland, and Weil (1997, 2000) in their analysis employing time non-separable preferences. Given the restriction for condition (ii) of Assumption 1 to hold, this function violates concavity if and only if $\rho < 0$. Alonso-Carrera, Caballé, and Raurich (2003) establish conditions guaranteeing the existence of an interior solution, even when concavity is violated. These include the condition $\gamma > 1$ (intertemporal elasticity of substitution less than unity), a condition that empirical evidence strongly supports.

[30] Early work by Frankel (1962) also proposed a production function of the general form of (29).

consumption, capital, and output all growing at the same constant rate, yielding a constant consumption-capital ratio. Let ψ^* denote the common steady-state growth rate for the decentralized economy, and $\tilde{\psi}$ the steady state growth rate of the planner's economy. It is straightforward to derive the growth rates and the consumption-capital ratios of the two economies:[31]

Decentralized economy:

$$\psi^* = \frac{A\alpha - \beta}{1-(1-\gamma)(1+\rho)} \tag{30a}$$

$$\frac{C^*}{K^*} = \frac{\beta - A(1-\gamma)(1+\rho) + A(1-\alpha)}{1-(1-\gamma)(1+\rho)} \tag{30b}$$

Centrally planned economy

$$\tilde{\psi} = \frac{A - \beta}{1-(1-\gamma)(1+\rho)} \tag{31a}$$

$$\frac{\tilde{C}}{\tilde{K}} = \frac{\beta - A(1-\gamma)(1+\rho)}{1-(1-\gamma)(1+\rho)} \tag{31b}$$

Subtracting equations (31) from (32) we obtain

$$\psi^* - \tilde{\psi} = -\frac{A(1-\alpha)}{1-(1-\gamma)(1+\rho)} \le 0 \tag{32a}$$

$$\frac{C^*}{K^*} - \frac{\tilde{C}}{\tilde{K}} = \frac{A(1-\alpha)}{1-(1-\gamma)(1+\rho)} \ge 0 \tag{32b}$$

Because the utility function in (28) satisfies (13), deviations from the optimum occur only in the presence of production externalities. If they are positive, the decentralized economy accumulates too little capital, and consumes too much; consequently its growth rate is too low. Consumption externalities alone do not cause distortions but they do influence the magnitude of the distortions caused by the production externalities. Thus,

[31] For the more general homogenous production function, the parameter α is replaced the equilibrium marginal physical production of private capital.

$$\frac{\partial(\psi^* - \tilde{\psi})}{\partial\rho} = -\frac{A(1-\alpha)(1-\gamma)}{\left[1-(1-\gamma)(1+\rho)\right]^2} > 0 \text{ if } \gamma > 1 \tag{33}$$

so that an increase in the degree of jealousy will increase the gap between the actual and optimal equilibrium growth rates if $\gamma > 1$, i.e. the intertemporal elasticity of substitution is less than one. Similarly, with $\gamma > 1$ an increase in the degree of jealousy will increase the distortion from the production externality on the equilibrium consumption-capital ratio. We may thus state

Proposition 7: *The symmetric equilibrium of the endogenous growth model with fixed labor supply has the following properties:*

1. *The equilibrium growth rate is below its optimum ($\psi^* < \tilde{\psi}$) and the equilibrium consumption-capital ratio is higher than its optimum ($C^*/K^* > \tilde{C}/\tilde{K}$) if and only if there is a positive production externality. Consumption externalities alone do not cause the equilibrium growth rate to deviate from its optimum.*
2. *If the intertemporal elasticity of substitution is less than one, then an increase in the magnitude of negative consumption externality (more jealousy) will exacerbate the distortion created by the positive production externality.*

As is well documented in the endogenous growth literature, subsidizing capital at the rate

$$\tau_k = -\frac{1-\alpha}{\alpha} \tag{34}$$

will replicate the first-best optimum, irrespective of whether or not consumption externalities exist.[32]

6.2 Elastic Labor Supply

With endogenous labor supply, the preferences of the representative agent now become:

$$\int_0^\infty \frac{1}{1-\gamma}[cC^\rho(1-L)^\phi]^{1-\gamma} e^{-\beta t} dt \tag{35}$$

where $\phi > 0$. Following Romer (1986) and others the agent's production function is modified to:

[32] This is identical to the condition $\tau_k = -F_K/F_k$ obtained for the stationary Ramsey model.

$$\dot{k} = Ak^{\alpha}L^{\delta}K^{1-\alpha} - c \tag{36}$$

which is constant returns in individual capital, k, and aggregate capital K.[33] The symmetric equilibria in the decentralized and centrally planned economies are now respectively

Decentralized economy:

$$\psi^* = \frac{(\alpha AL^{*\delta} - \beta)}{1-(1-\gamma)(1+\rho)} \tag{37a}$$

$$\psi^* = AL^{*\delta} - \frac{C^*}{K^*} \tag{37b}$$

$$\frac{C^*}{K^*} = \frac{\delta A}{\phi}(1-L^*)L^{*(\delta-1)} \tag{37c}$$

Centrally planned economy

$$\tilde{\psi} = \frac{(A\tilde{L}^{\delta} - \beta)}{1-(1-\gamma)(1+\rho)} \tag{38a}$$

$$\tilde{\psi} = A\tilde{L}^{\delta} - \frac{\tilde{C}}{\tilde{K}} \tag{38b}$$

$$\frac{\tilde{C}}{\tilde{K}} = \frac{(1+\rho)\delta A}{\phi}(1-\tilde{L})\tilde{L}^{(\delta-1)} \tag{38c}$$

Comparing the growth rates of the two economies, given by (37a) and (38a) respectively, the difference comes from two factors. The first is the undervaluation of the social marginal product of capital in the decentralized economy in the case of a positive production externality. The second is the differential labor supply $(L^*\ vs.\ \tilde{L})$, in the presence of the consumption and capital externalities. To see the difference in the growth rates and other aspects more closely, we apply Taylor expansions to the right hand sides of equations (37a) – (37c) around the efficient allocation ($\tilde{\psi}, \tilde{L}, \tilde{C}/\tilde{K}$). The

[33] In order to keep the consumption externality, which operates through the labor-leisure choice, distinct from the production externality, we do not impose the condition $\alpha + \delta = 1$. In the case that $\delta + \alpha < 1$ we assume that the residual output is the form of rent accruing to a fixed factor, land say; if $\delta + \alpha > 1$ we assume that labor is paid its marginal product, with the remaining output going to capital. In this case (37a) would be modified to $\psi^* = ((1-\delta)AL^{*\delta} - \beta)/(1-(1-\gamma)(1+\rho))$. We view this issue as a technicality that has little bearing on our conclusions.

details are provided in the Appendix and the results are summarized in the following proposition:

Proposition 8: *Suppose that the intertemporal elasticity of substitution is less than one. The symmetric equilibrium of the endogenous growth model with endogenous labor supply has the following properties:*

1. *In the case of a negative consumption externality (i.e. preferences exhibit jealousy) without production externalities, the equilibrium growth rate, labor supply, and consumption-capital ratio are all above their optimal levels, i.e. (*$\psi^* > \tilde{\psi}$, $L^* > \tilde{L}$, $C^*/K^* > \tilde{C}/\tilde{K}$*). In the case of admiration, these relationships are reversed.*

2. *In the case of a positive production externality without consumption externalities, the equilibrium growth rate and labor supply are both below their optimal levels, i.e. (*$\psi^* < \tilde{\psi}$, $L^* < \tilde{L}$*), while the equilibrium consumption-capital ratio is above the efficient level i.e. (*$C^*/K^* > \tilde{C}/\tilde{K}$*).*

Analogous to the result in Section 6.1, positive production externalities result in a capital ratio that is too low for the decentralized economy. But here with endogenous labor supply, equilibrium employment is affected by both the consumption and production externalities. Given the production function, (36), capital and labor input are complements. Positive production externalities cause an under-investment in private capital, and thus result in a labor supply that is too low. Both of these effects lower the equilibrium growth rate and induce the agent to over-consume. On the other hand, jealousy results in an over-valuation of consumption relative to leisure. It therefore generates too much labor supply (or too little leisure). As a consequence the equilibrium growth rate for the decentralized economy is too high. Admiration will produce the opposite effect.

As in the previous cases the distortions created by the externalities in the decentralized economy can be corrected by taxation. Introducing taxes, (37a) and (37c) are modified to

$$\psi^* = \frac{(\alpha(1-\tau_k)AL^{*\delta} - \beta)}{1-(1-\gamma)(1+\rho)} \tag{37a'}$$

$$\frac{C^*}{K^*} = \left(\frac{1-\tau_w}{1+\tau_c}\right)\frac{\delta A}{\phi}(1-L^*)L^{*(\delta-1)} \tag{37c'}$$

By direct comparison with (38a) and (38c), setting

$$\alpha(1-\tau_k)=1 \tag{39a}$$

$$\frac{1-\tau_w}{1+\tau_c}=1+\rho \tag{39b}$$

replicates the first-best optimum.[34] Equation (39a) is equivalent to (34), which therefore holds whether or not labor supply is elastic. Equation (39b) is analogous to (27) and asserts that any combination of a tax on labor income and on consumption consistent with that condition will correct for the consumption externality. For example, setting $\tau_c = 0$, (39b) implies $\tau_w = -\rho$; i.e. a subsidy to (tax on) labor income equal to the degree of admiration (jealousy). But the same can be accomplished by subsidizing or taxing consumption.

7. Conclusions

This paper has examined the effects of consumption and production externalities on the welfare of a growing economy and discussed the appropriate corrective taxes. Both types of externalities have been shown to have significant consequences for resource allocation and a number of important conclusions can be drawn. First, externalities as expressed by the first derivatives of the utility and production functions with respect to aggregate consumption and capital, respectively, are the critical determinants of the qualitative effects of these externalities. The impacts of externalities as described by cross partial derivatives, and embodied in the notions of keeping up with the Joneses in the case of consumption externalities, are less important in this regard, although they are important in determining the magnitudes of these effects. Second, production externalities will affect the long-run equilibrium irrespective of the endogeneity of labor supply, whereas consumption externalities will affect the long-run equilibrium if and only if the labor supply is elastic. This is because consumption externalities affect the long-run equilibrium through the tradeoff between consumption and leisure in utility, together with the interaction of labor and capital in production. The elasticity of labor supply thus becomes an important determinant of the empirical

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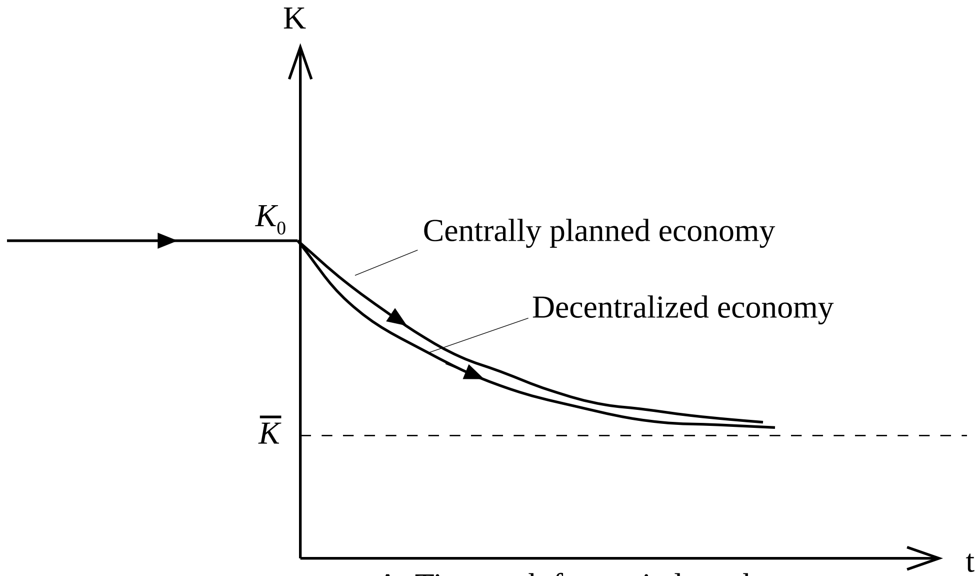
[34] These conditions are analogous to those obtained by Turnovsky (2000)

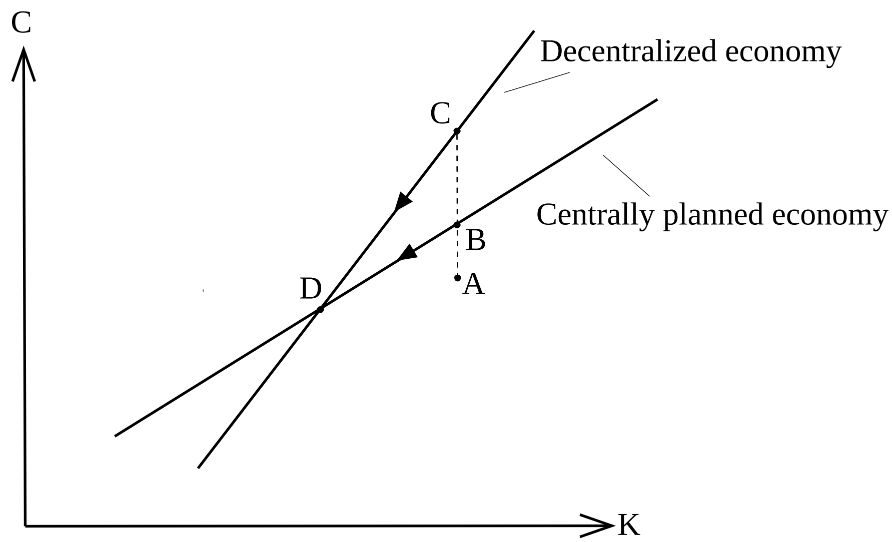
significance of consumption externalities. With endogenous labor, negative consumption externalities lead to a steady state with an over-supply of both capital and labor, and over-consumption. Although with fixed labor supply, consumption externalities have no long-run effects, they nevertheless may have distortionary effects along the transitional path, depending upon the form of the utility function. Third, we suggest a tax structure, and derive optimal tax rates, to correct for the distortions created by the externalities. In general we find that in the stationary economy, a combination of an appropriate consumption tax (or subsidy) to correct for the consumption externality, together with a capital income tax (or subsidy) to correct for the production distortion, can replicate the social optimum. Moreover, externalities of a plausible yet modest magnitude may require quite large corrective taxes (or subsidies).

We have considered both stationary and endogenously growing economies, and while there are many parallels in how externalities impact, there are also important differences. For example, whereas a positive production externality always leads to the long-run under-accumulation of capital and under-consumption in a stationary economy, it leads to over-consumption and thus to a sub-optimally slow growth rate in an endogenously growing economy. The contrast in how consumption and production externalities interact in the two economies is also interesting. With exogenous labor, for example, consumption externalities are irrelevant in the determination of the steady state in the stationary Ramsey model. In contrast, consumption externalities influence the gap between the equilibrium and optimal growth rates arising from a production externality in a growing economy.

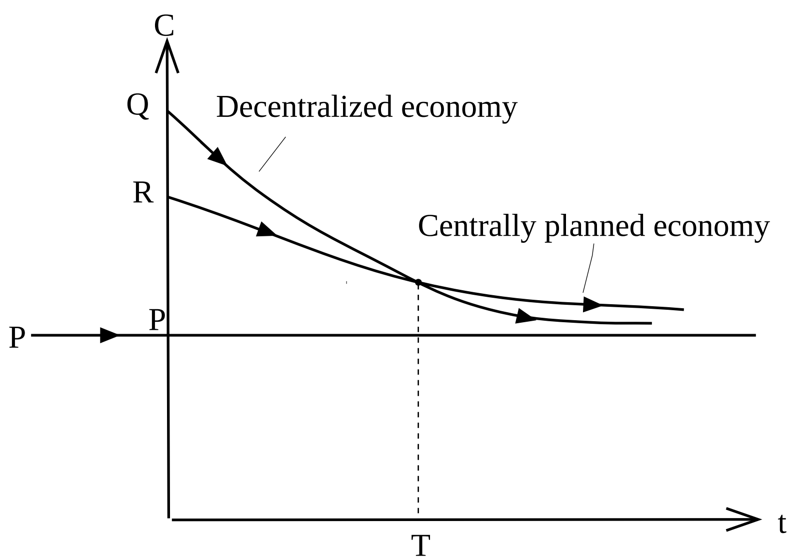
Finally, like the overwhelming bulk of the literature, this paper has been restricted to a closed economy. In Turnovsky and Liu (2004) we have extended our analysis of consumption and production externalities in the Ramsey model to a small open economy, having access to a perfect world financial market. International capital mobility increases the ways agents may respond to externalities, and as a result several of the propositions describing the impact of externalities require substantial modification when applied to a small open economy. Clearly, the issue of externalities in an international economy merits further investigation, although work in this direction is beginning; see also Fisher (2004).



A. Time path for capital stocks



B. Saddle paths for decentralized and centrally planned economies



C. Time paths for consumption in decentralized and centrally planned economies

Fig 1: Dynamic Adjustment in Presence of Consumption Externalities: $(\theta_d > \theta_p)$

Appendix:

Proof of proposition 2

Applying Taylor expansions to equations (6a) - (6d) around the optimal steady state ($\widetilde{C},\widetilde{K},\widetilde{L},\widetilde{\lambda}$) and rearranging the terms, we obtain the following:

$$\begin{bmatrix} U_{cc}+U_{cC} & -U_{cl} & 0 & -1 \\ U_{lc}+U_{lC} & -(U_{ll}+F_{LL}\tilde{\lambda}) & -(F_{Lk}+F_{LK})\tilde{\lambda} & -F_L \\ 0 & F_{kL} & F_{kk}+F_{kK} & 0 \\ -1 & F_L & \beta & 0 \end{bmatrix} \cdot \begin{bmatrix} C^*-\tilde{C} \\ L^*-\tilde{L} \\ K^*-\tilde{K} \\ \lambda^*-\tilde{\lambda} \end{bmatrix} = \begin{bmatrix} U_C \\ 0 \\ F_K \\ 0 \end{bmatrix} \qquad \text{(A.1)}$$

Let D denote the determinant of the matrix on the left hand side of equation (A.1). It can be shown that $\mathrm{D}>0$. Solving for $C^*-\widetilde{C}$, $K^*-\widetilde{K}$, and $L^*-\tilde{L}$, and we obtain

$$C^*-\tilde{C}=\frac{U_C}{D}[F_L^2(F_{kk}+F_{kK})-\beta F_L F_{kL}]+\frac{F_K}{D}[-F_L\tilde{\lambda}(F_{Lk}+F_{LK})-F_L\beta U_{cl}+(U_{ll}+F_{LL}\tilde{\lambda})\beta] \quad \text{(A.2a)}$$

$$K^*-\tilde{K}=\frac{(-F_{kL}F_L)U_C}{D}+\frac{F_K}{D}[-F_L U_{cl}-(U_{lc}+U_{lC})F_L+(U_{ll}+F_{LL}\tilde{\lambda})+F_L^2(U_{cc}+U_{cC})] \qquad \text{(A.2b)}$$

$$L^*-\tilde{L}=\frac{U_C}{D}[F_L(F_{kk}+F_{kK})]+\frac{F_K}{D}[(U_{cl}+U_{Cl})\beta-\tilde{\lambda}(F_{Lk}+F_{LK})-F_L\beta(U_{cc}+U_{cC})] \qquad \text{(A.2c)}$$

Note that the coefficients of U_C for all three equations (A.2a)-(A.2c) are negative. As a result, admiration ($U_C>0$) implies that consumption, capital and labor input are all too low in equilibrium. Similarly, the coefficients of F_K for equations (A.2a) and (A.2b) are also negative. Positive production externality from capital therefore causes that consumption and capital are too low in equilibrium. The coefficient of F_K on equation (A.2c) has an ambiguous sign. No definite conclusion can be made for the labor input level from the effect of production externalities.

Proof of Proposition 8

Applying Taylor expansions to equations (37a) - (37c) around the optimal steady state (38a) – (38c) and rearranging the terms, we obtain the following:

$$\begin{bmatrix} 1 & \dfrac{-\alpha\delta AL^{(\delta-1)}}{1-(1-\gamma)(1+\rho)} & 0 \\ 1 & -\delta AL^{(\delta-1)} & 1 \\ 0 & \dfrac{-\delta AL^{(\delta-2)}}{\phi}((1-\delta)+\delta L) & -1 \end{bmatrix} \begin{bmatrix} \psi^* - \tilde{\psi} \\ L^* - \tilde{L} \\ \dfrac{C^*}{K^*} - \dfrac{\tilde{C}}{\tilde{K}} \end{bmatrix} = \begin{bmatrix} \dfrac{-(1-\alpha)AL^{\delta}}{1-(1-\gamma)(1+\rho)} \\ 0 \\ \dfrac{\delta\rho A}{\phi}(1-\tilde{L})\tilde{L}^{(\delta-1)} \end{bmatrix} \tag{A.3}$$

Let D denote the determinant of the matrix on the left hand side of (A.3). Then it can be shown that

$$D = \delta AL^{(\delta-1)}\left(\frac{(1-\alpha-(1-\gamma)(1+\rho))}{1-(1-\gamma)(1+\rho)} + \frac{((1-\delta)L^{-1}+\delta)}{\phi}\right) \tag{A.4}$$

and $D > 0$ for $\gamma \geq 1$. From (A.3) we can summarize the effects of the consumption and production externalities as follows

$$\psi^* - \tilde{\psi} = \frac{1}{D}\left\{\frac{-(1-\alpha)AL^{\delta}}{1-(1-\gamma)(1+\rho)}\left[\delta AL^{\delta-1} + \frac{\delta A}{\phi}((1-\delta)+\delta L)L^{\delta-2}\right] - \frac{\rho\alpha\delta^2 A^2 L^{2(\delta-1)}}{\phi}\frac{(1-L)}{1-\gamma(1+\rho)}\right\} \tag{A.5a}$$

$$L^* - \tilde{L} = \frac{1}{D}\left\{\frac{-(1-\alpha)AL^{\delta}}{1-(1-\gamma)(1+\rho)} - \frac{\rho\delta A}{\phi}(1-L)L^{\delta-1}\right\} \tag{A.5b}$$

$$\frac{C^*}{K^*} - \frac{\tilde{C}}{\tilde{K}} = \frac{1}{D}\left\{(1-\alpha)A^2L^{2(\delta-1)}\frac{\delta}{\phi}((1-\delta)+\delta L) - \frac{\rho\delta^2 A^2}{\phi}(1-L)L^{2(\delta-1)}\left[\frac{1-\alpha-(1-\gamma)(1+\rho)}{1-(1-\gamma)(1+\rho)}\right]\right\} \tag{A.5c}$$

Note that the braces on the right hand sides of all three equations (A.5a)-(A.5c) each contain two terms. The first reflect the production externalities. The signs of these terms are determined by $(1-\alpha)$ and equal zero in the absence of these effects $(1-\alpha = 0)$. The second terms reflect the consumption externalities. Their signs are determined by ρ and equal zero in the absence of such externalities ($\rho = 0$). Thus, we have

1. In the case of positive production externalities without any consumption externalities;

$$sgn(\psi^* - \tilde{\psi}) = sgn(L^* - \tilde{L}) = sgn\left(\frac{\tilde{C}}{\tilde{K}} - \frac{C^*}{K^*}\right) = -sgn(1-\alpha) < 0$$

2. In the case of positive consumption externalities without any production externalities,

$$sgn(\psi^* - \tilde{\psi}) = sgn(L^* - \tilde{L}) = sgn\left(\frac{C^*}{K^*} - \frac{\tilde{C}}{\tilde{K}}\right) = -sgn(\rho) < 0.$$

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