

**Intergenerational Allocation of Government Expenditures:  
Externalities and Optimal Taxation\***

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**Abstract:**

This paper studies optimal capital and labor income taxes when the benefits of public goods are age-dependent. Provided the government can impose a consumption tax, it can attain the first-best resource allocation. This involves the uniform taxation of the cohorts' labor income and a zero capital income tax. With no consumption tax and optimally chosen government spending, labor income should be taxed non-uniformly across cohorts and the capital income tax should be non zero. Deviations of the public goods from their respective optima create distortions. These affect the labor supply decisions of both cohorts and capital accumulation, providing a further reason to tax (or subsidize) capital income.

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## 1. Introduction

The allocation of public expenditure is highly skewed toward the older members of society. In 1997, the U.S. government spent \$27,400 per person on people of over 85 years, and \$21,817 on people in the 75-84 age bracket. In contrast, it spent only \$4526 on the young of the 0-19 age group, and \$2256 per capita on the 20-34 age group.<sup>1</sup> A similar pattern of public expenditures is also observed in other OECD countries. For example, in the U.K. per capita public spending on pensioners in 2001-02 was £7520 compared to £4900 for children.<sup>2</sup> These examples suggest the presence of a profound cohort bias in the allocation of public expenditures. This raises (at least) two important questions. First, is the observed pattern of public expenditure allocation optimal or even socially desirable, and second, what are consequences for taxation policy in general and for the optimal tax structure in particular?

This paper investigates these issues, paying particular attention to the problem of optimal capital income taxation in the context of the public provision of goods and services, the benefits of which are age-dependent. In doing so, it builds on the important contributions of Atkeson, Chari, and Kehoe (1999), Chari and Kehoe (1999), and Erosa and Gervais (2001, 2002) who address the issue of optimal taxation in life-cycle economies.<sup>3</sup> In all of these contributions the primary focus is on the optimal tax structure, with the public good playing only a minor role as a spending item in the government's budget. Our paper attributes an essential role to government expenditures, stressing the qualitative impacts on the different cohorts, and their consequences for tax policy.

Despite its importance, the allocation of government spending on different cohorts and its implications for tax policy has received little attention by economists. One strand of literature, known as Generational Accounting, pioneered by Auerbach, Gokhale, and Kotlikoff (1991), is close to the spirit of this work. It studies how much government transfers to each generation and how much each generation pays to the government, and the consequences on the tax burden of future

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<sup>1</sup> Urban Institute. These figures relate to all levels of government.

<sup>2</sup> See Sefton (2004).

<sup>3</sup> They also consider infinite-lived agents, emphasizing the contrasting effects of the two economic structures on the optimal tax structure.

generations. The central focus of this literature is on the intertemporal solvency of the government, an issue that is also relevant here. On the other hand, the political economy literature focuses mostly on the allocation process as competition for public resources between cohorts; see e.g. Persson and Tabellini (2000) and Tabellini (1991). The focus of this literature is on the political economy equilibrium determined by political forces such as voters, lobbying groups, etc., rather than on the economic consequences of the allocation, which is the central aspect of our analysis. In contrast, the present paper addresses the question of the optimal tax structure in the context of optimally provided public goods in a life cycle economy. Our particular concern is to investigate the relationship between the provision of cohort-specific public goods and optimal tax policy.

To this end we employ a two period overlapping generations model in which public spending is utility-enhancing for both cohorts, and thus interacts with their respective consumption and labor supply decisions.<sup>4</sup> The government can finance its expenditures either by imposing distortionary taxes or by issuing debt, subject to its intertemporal solvency. The issue of the optimal tax structure dates back to Ramsey (1927) and we adopt the so-called “primal approach” to this problem, whereby the government selects the allocation of resources directly, subject to an “implementability constraint” imposed upon by the behavior of the private sector, in response to the tax rates it faces.

A key feature of our analysis is that the government sets its expenditures on each cohort as specified fractions of output. This is a plausible specification of spending policy in a growing economy; see e.g. Barro (1990), Futagami et al. (1993), and Turnovsky (1996).<sup>5</sup> We approach the analysis in two stages, the first where the government sets its expenditure shares arbitrarily, and second where it does so optimally. There are two good reasons for proceeding in this sequential manner. The first is pedagogic. With government expenditures specified as shares, an increase in any productive factor increases total output, and thus the level of government spending on either cohort, thereby creating potential externalities. For arbitrarily set spending shares, these externalities may produce distortions at both the inter and intratemporal margins, although these distortions disappear when government spending is set optimally. In this context, this sequential approach

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<sup>4</sup> The same approach can be applied to public goods that are productive as in the infinitely-lived representative agent models of Barro (1990) and Turnovsky (1996), for example.

<sup>5</sup> In some cases, the expenditure share is specified exogenously, while in others it arises as a consequence of imposing a balanced budget on the government.

enhances our understanding of optimal government spending policy. The second reason is more practical. Changing government spending programs is notoriously slow, typically involving lengthy legislation, and governments are inevitably adjusting to expenditure levels that at one time may have been optimal, but are no longer so.

Our results are summarized in several propositions. We first show that as long as the government has at its disposal a consumption tax, it can attain the first-best allocation of resources, as chosen by a central planner. This involves the uniform taxation of labor across cohorts, offset by an equal consumption subsidy, and a zero tax on capital.

The absence of a consumption tax imposes severe constraints. In the case that government expenditure on each cohort is chosen optimally, the labor income of the differential cohorts should in general be taxed differentially, and the tax on capital income should be non-zero, thus extending the results of Erosa and Gervais (2001, 2002). For the widely employed constant elasticity utility function the labor income of both cohorts should be taxed positively, with the cohort spending the longer work time being taxed more heavily. The tax on capital should be positive if and only if the leisure time of the young exceeds that of the old.

The more interesting case arises when the public good is not provided at the socially optimal level. As just noted, this deviation from the optimal level creates distortions in the economy and affects the labor supply and capital accumulation decisions of young and old. If the government allocates too much (insufficient) output to either cohort, then the labor income of both cohorts should be taxed at above (below) the optimal rate and the tax on capital income should be positive (negative). Moreover, in the case that the overall fraction of government spending is optimal, but is incorrectly allocated to cohorts, the first-best optimal tax structure still applies.

The seminal studies of Judd (1985) and Chamley (1986) arguing that the long-run tax on capital income should be zero has served as a benchmark for much of the literature on optimal capital income taxation. The question of the robustness of this result has received much attention and indeed several authors in various contexts have shown how this result need not always apply; see e.g. Correia (1996), Turnovsky (1996, 2000), Jones et al. (1997), Cremer et al (2003) and García-Peñalosa and Turnovsky (2005). Using a life cycle model, Erosa and Gervais (2002) showed

that as long as certain key “general equilibrium expenditure elasticities”, vary with the life cycle, [which are otherwise constant in a representative agent model] it is not optimal to leave capital income untaxed. However, this result is also sensitive to the choice of preferences. This study shows that regardless of the choice of preferences, if the public provision is not optimal for the age-cohorts, any deviation from the optimal in the allocation process will result in a non zero capital income tax.<sup>6</sup>

The organization of this paper is as follows. Section 2 sets out the analytical framework, while Section 3 derives the first-best outcome in a centrally planned economy. Section 4 sets out the optimal fiscal policy (optimal Ramsey problem) in a decentralized economy, while Section 5 provides a detailed characterization of optimal steady-state fiscal policy. Section 6 briefly summarizes, while a short appendix provides some technical details.

## 2. The Analytical Framework

We consider an overlapping generations (OLG) model in which agents live for two periods. Thus, at any point of time,  $t$  say, there are two cohorts: young (1) and old (2). We abstract from population growth, so that the sizes of both cohorts are the same, normalized at unity.<sup>7</sup>

Both young and old are endowed with a unit of time and supply labor elastically,  $0 < l_{1,t} < 1$ ,  $0 < l_{2,t} < 1$ . In return they each earn a market determined wage rate,  $w_{1,t}$ ,  $w_{2,t}$  which is taxed at the proportional rate  $\tau_1^w$  and  $\tau_2^w$ , respectively<sup>8</sup>. While the assumption that the tax on labor income is cohort-specific may seem unusual, it is in fact not unreasonable to assume that tax rates on labor income vary across cohorts. Indeed, if tax rates increase with income and if income increases with age, Gervais (2004) has argued that with little progressivity in the tax rates, age-dependent tax rates come close to reflecting the actual progressive tax rates<sup>9</sup>. Moreover, this is compounded by the presence of differential deductions and exemptions available to tax payers at different stages of their

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<sup>6</sup> This result is analogous to findings in the infinitely-lived representative agent model; see e.g. Turnovsky (1996).

<sup>7</sup> We assume that all agents within a cohort are homogeneous, having the same ability.

<sup>8</sup> For a discussion of the OLG model with endogenous labor supply, see Ordober and Phelps (1975), Park (1991), Nourry (2001), Michel and Pestieau (2004), where labor supply is endogenous only in the first period; Crettez and Le Maitre (2002) where labor supply is endogenous only in the second period; and Gaumont and Leonard (2003) where labor is supplied endogenously in both periods.

<sup>9</sup> He compared the optimal tax rates derived from life cycle model with the average tax rates paid by the individual in US. He found that optimal tax rates are comparable with the age profile of the labor income taxes given by US tax code.

life-cycle.<sup>10</sup> As a further rationale, we may note that since one of our main objectives is to study the optimal tax structure, the flexibility of cohort-specific wage income taxes may be necessary to help achieve the optimum.<sup>11</sup> The capital in the economy is owned by the old. It yields a rate of return,  $r_t$ , and is subject to tax,  $\tau^k$ .

## 2.1 Firms

We assume that output is produced competitively in accordance with the aggregate neoclassical production function

$$Y_t = F(K_t, l_{1,t}, l_{2,t}) \quad (1a)$$

Both young labor and old labor, as well as capital, are used for production, with the factor returns being determined by their respective marginal products

$$r_t = F_K(K, l_{1,t}, l_{2,t}); w_{1,t} = F_{l_{1,t}}(K, l_{1,t}, l_{2,t}); w_{2,t} = F_{l_{2,t}}(K, l_{1,t}, l_{2,t}) \quad (1b)$$

## 2.2 Households

Consider an agent born at time  $t$  and living for two periods. During the first period he earns income from labor taxed at the rate  $\tau_{1,t}^w$ , and uses the proceeds to buy capital,  $K_{t+1}$ , government bonds,  $B_{t+1}$ , and consumption, which is taxed at a rate  $\tau_t^c$  that is uniform across cohorts. The following period, when he is old, the agent consumes the after-tax proceeds of capital, bonds, and labor income. Each period the government provides two goods,  $G_{1,t}$ , which benefits the young, and  $G_{2,t}$ , which benefits the old.

Thus the agent chooses his allocation of consumption,  $C_{1,t}, C_{2,t}$  and labor supply by solving the following optimization problem defined over his two-period lifetime:

$$\text{Max } W_t \equiv U^1(C_{1,t}, l_{1,t}, G_{1,t}) + \beta U^2(C_{2,t+1}, l_{2,t+1}, G_{2,t+1}) \quad \text{where } 0 < \beta < 1 \quad (2a)$$

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<sup>10</sup> These are things like deductions for education and medical expenses that tend to vary with age.

<sup>11</sup> As we shall observe in footnote 16 below, cohort-neutral wage income tax rates will further constrain the choice of the optimal tax structure.

subject to

$$(1 + \tau_t^c)C_{1,t} + K_{t+1} + B_{t+1} = w_{1,t}l_{1,t}[1 - \tau_{1,t}^w] \quad (2b)$$

$$(1 + \tau_{t+1}^c)C_{2,t+1} = w_{2,t+1}l_{2,t+1}[1 - \tau_{2,t+1}^w] + (K_{t+1} + B_{t+1})[1 + r_{t+1}(1 - \tau_{t+1}^k)] \quad (2c)$$

where  $\tau_{t+1}^k$  is the tax on capital income earned by the old generation. Observe that we assume that government bonds yield the same return as capital and both are taxed at a common rate.

The agent is assumed to derive positive marginal utility from private consumption and the public good, and disutility from labor.<sup>12</sup> The assumption that  $G_1$  benefits only the young, while  $G_2$  benefits only the old is a polar one, but serves as a convenient benchmark. Education is an example of the former, and medical care is an example of the latter. It is true that having educated young and healthy old may also benefit the other cohort, through enhanced productivity, for example. Furthermore, many forms of government spending, like national defense or environmental control, are cohort-neutral, while others, such as highways, are likely to favor the young more through their greater usage. Moreover, since the agents may have entirely different utility functions over the two periods of their life cycle, we may also interpret the public good as being a single non-excludable public good that yields differential benefits to the two groups, such as the highway example, just cited.

We assume that the agent, in making his private allocation decisions takes the level of government spending as given. Therefore, optimizing (2a), subject to (2b) and (2c), with respect to  $C_{1,t}, l_{1,t}, C_{2,t}, l_{2,t}$ , and wealth,  $K_{t+1} + B_{t+1}$  implies the following optimality conditions:

$$-\frac{U^1_{l_{1,t}}}{U^1_{C_{1,t}}} = w_{1,t} \left[ \frac{1 - \tau_{1,t}^w}{1 + \tau_t^c} \right] \quad (3a)$$

$$-\frac{U^2_{l_{2,t+1}}}{U^2_{C_{2,t+1}}} = w_{2,t+1} \left[ \frac{1 - \tau_{2,t+1}^w}{1 + \tau_{t+1}^c} \right] \quad (3b)$$

$$\frac{U^1_{C_{1,t}}}{U^2_{C_{2,t+1}}} = \left[ \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \right] \beta [1 + r_{t+1}(1 - \tau_{t+1}^k)] \quad (3c)$$

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<sup>12</sup> That is we assume:  $U_{C_i}^i > 0, U_{l_i}^i < 0, U_{G_i}^i > 0$ . It is also concave in  $C_i, l_i$  for  $i = 1, 2$ .

Equations (3a) and (3b) are standard static efficiency conditions, equating the marginal rate of substitution between labor and consumption during the two periods [young and old] to their respective after-tax wage rates, adjusted for the consumption tax. Equation (3) is the intertemporal efficiency condition that equates the marginal rate of substitution of consumption between young and old to the tax-adjusted, discounted, rate of return on savings. These three optimality conditions, together with the budget constraints (2b) and (2c), can be solved for  $C_{1,t}, C_{2,t+1}, l_{1,t}, l_{2,t+1}$ , and  $(K_{t+1} + B_{t+1})$  as function of wage rates, rental rate of capital, and the fiscal policy variables.

### 2.3 Government budget constraint

We assume that the government conducts its expenditures subject to its budget constraint

$$B_{t+1} = (1 + r_t)B_t + G_{1,t} + G_{2,t} - \tau_{1,t}^w w_{1,t} l_{1,t} - \tau_{2,t}^w w_{2,t} l_{2,t} - \tau_t^c (C_{1,t} + C_{2,t}) - \tau_t^k r_t (K_t + B_t) \quad (4)$$

This equation asserts that the government sells bonds to finance the imbalance between its expenditures on (i) the young and the old and (ii) interests payment on its outstanding debt at time  $t$ , and the total revenues collected from (i) the wage tax, (ii) the consumption tax, and (iii) the tax on asset income.

A key issue concerns how governments determine their spending allocations. With capital being accumulated, this economy evolves over time, and in practice it is plausible for such economies to set spending as a fraction of output. That is, we assume:

$$G_{1,t} = g_{1,t} Y_t ; G_{2,t} = g_{2,t} Y_t \quad (5)$$

This specification endogenizes the level of spending, just as it does tax collection. It is important to note that tying government spending to output in this way introduces two types of externalities – *within-cohort* externalities and *cross-cohort* externalities. Within-cohort externalities capture the fact that the labor supply or capital accumulation decision of an individual from a cohort benefits all who belong to that cohort. For example, an increase in a young person's labor supply expands total output and thus the total amount spent on them all. Since government spending is utility enhancing, an increase in labor supply of a young increases the utility of all young. At the same time, the

expansion of output also increases the total expenditure on the old cohort and their utility. This is the cross-cohort externality.<sup>13</sup>

By taking government spending as given, agents do not take these externalities into account when making their individual labor supply and capital accumulation decisions in a decentralized economy. Each individual perceives that his labor supply or capital accumulation decisions are too insignificant to influence the total output and the government spending on cohorts. We shall show that when government spending is allocated optimally these cohort externalities disappear.

## 2.4 Economy-wide Resource Constraint

Summing over the two cohorts in existence at time  $t$ , and the government budget constraint, and using the linear homogeneity of the technology, we immediately obtain the economy-wide resource constraint

$$K_{t+1} - K_t = F(K_t, l_{1,t}, l_{2,t}) - C_{1,t} - C_{2,t} - G_{1,t} - G_{2,t} \quad (6)$$

which we can express in the equivalent form

$$K_{t+1} - K_t = (1 - g_{1,t} - g_{2,t})F(K_t, l_{1,t}, l_{2,t}) - C_{1,t} - C_{2,t} \quad (6')$$

## 3. First-best outcome for centrally planned economy

As a benchmark, it is instructive to set out briefly the steady-state first best optimal outcome that a central planner, having direct control over resources would achieve. To do this, we need to introduce a welfare function. Because of the heterogeneity of agents, this choice is not as natural as in the conventional infinitely-lived representative agent model. We follow tradition and assume that social welfare is the discounted sum of lifetime welfares of the different cohorts, namely

$$\delta^{-1}U_0^2 + \sum_{t=0}^{\infty} \delta^t W_t \quad (7)$$

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<sup>13</sup> This notion of cross-cohort externalities is close in spirit to the idea of “fiscal leakage” from one generation to another as discussed by Razin, Sadka, and Swagel (2002), who examined the effect of aging on tax policies. When the share of old people in total population increases, total capital tax revenue also increases and if this revenue is distributed among the young, there is a cross-generational transfer.

where  $0 < \delta < 1$  is the intergenerational discount factor,  $W_t$  is the intertemporal welfare of the generation born at time  $t$ , and  $U_0^2$  is the second-period utility of the agents born prior to the planning horizon. The discount rate is taken to be exogenous and the assumption  $\delta$  less than one implies declining weight on successive generations.

The central planner is assumed to choose the quantities  $C_{1,t}, C_{2,t}, l_{1,t}, l_{2,t}, K_t, g_{1,t}, g_{2,t}$  to maximize (7) subject to the resource constraint (6'). Omitting details, it is straightforward to show that the resulting first-best steady state will be characterized by the following conditions

$$U_c^1(\tilde{C}_1, \tilde{l}_1, G_1) = \tilde{\varphi} \quad (8a)$$

$$\frac{\beta}{\delta} U_c^2(\tilde{C}_2, \tilde{l}_2, G_2) = \tilde{\varphi} \quad (8b)$$

$$-U_l^1(\tilde{C}_1, \tilde{l}_1, G_1) = \tilde{\varphi} F_{l_1}(\tilde{K}, \tilde{l}_1, \tilde{l}_2) \quad (8c)$$

$$-\frac{\beta}{\delta} U_{l_2}^2(\tilde{C}_1, \tilde{l}_2, G_1) = \tilde{\varphi} F_{l_2}(\tilde{K}, \tilde{l}_1, \tilde{l}_2) \quad (8d)$$

$$F_K(\tilde{K}, \tilde{l}_1, \tilde{l}_2) = \frac{1-\delta}{\delta} \quad (8e)$$

$$F(\tilde{K}, \tilde{l}_1, \tilde{l}_2) = \tilde{C}_1 + \tilde{C}_2 + G_1 + G_2 \quad (8f)$$

$$G_1 = g_1 F(\tilde{K}, \tilde{l}_1, \tilde{l}_2) \quad (8g)$$

$$G_2 = g_2 F(\tilde{K}, \tilde{l}_1, \tilde{l}_2) \quad (8h)$$

$$U_G^1(C_1, l_1, G_1) = \tilde{\varphi} \quad (8i)$$

$$\frac{\beta}{\delta} U_G^2(C_2, l_2, G_2) = \tilde{\varphi} \quad (8j)$$

where  $\tilde{\varphi}$  is the shadow value associated with the economy-wide resource constraint. Equations (8a) – (8h) determine the steady-state values of  $\tilde{C}_1, \tilde{C}_2, \tilde{l}_1, \tilde{l}_2, \tilde{K}, \tilde{\varphi}, G_1, G_2$  for arbitrarily given values government expenditure shares,  $g_1, g_2$ , while the final two equations determine the optimal shares of government spending. These equations have straightforward interpretations; in particular (8a) – (8d)

and (8i), (8j) involve equating appropriate marginal utilities to shadow values. Because the central planner is choosing quantities directly and internalizes all externalities, the same optimum is reached whether he is choosing the shares,  $g_1, g_2$ , or the quantities,  $G_1, G_2$ , directly.

Equations (8) determine the overall first best steady-state equilibrium. Of particular interest is the optimal allocation of the public good between the young and the old. In general, this will depend upon all the parameters characterizing the economy. In the special and widely employed case that the utility function is additively separable in leisure, being of the form

$$W_t \equiv \frac{1}{\gamma} (C_{1,t} G_{1,t}^{\nu_1})^\gamma + V^1 (1 - l_{1,t}) + \beta \left[ \frac{1}{\gamma} (C_{2,t} G_{2,t}^{\nu_2})^\gamma + V^2 (1 - l_{2,t}) \right]$$

it can be easily shown that the optimal allocations of government spending to the two cohorts satisfies

$$\frac{G_2^{1-\gamma(1+\nu_2)}}{G_1^{1-\gamma(1+\nu_1)}} = \frac{\beta}{\delta} \left( \frac{\nu_2}{\nu_1} \right)^{1-\gamma}$$

Expenditure will be allocated more toward the old cohort, the larger  $\nu_2$  relative to  $\nu_1$  (the more they value government services), the larger  $\beta$ , (the less agents discount old age), and the smaller  $\delta$ , (the less the planner cares about future generations).<sup>14</sup> If  $\nu_2 = \nu_1$  so that both cohorts value their respective government services equally, then the government should spend more on the old ( $G_2 > G_1$ ) if and only if each agent discounts his old age less than the planner discounts future generations ( $\beta > \delta$ ).

#### 4. Optimal fiscal policy in the decentralized economy

We turn now to the main issue, namely the optimal composition of government spending and its financing in a decentralized economy. The nature of the problem and the ability of the decentralized economy to replicate the first-best optimum of the central planner depends upon the set of tax instruments available to the fiscal authority. With lump-sum taxation, the central planner can always replicate the first-best optimum, and hence the problem is only of interest if this possibility is

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<sup>14</sup> In the case of logarithmic utility  $G_2/G_1 = (\beta/\delta)(\nu_2/\nu_1)$

excluded, as is the case here.

As several authors have noted, this problem, which dates back to Ramsey (1927), can be conveniently studied by allowing the government to operate as a central planner and to pick the allocation directly, constrained by the optimal choices of agents in the decentralized economy, imposed by the fiscal structure.<sup>15</sup>

The key additional constraint that needs to be taken into account is the so-called “implementability constraint”. In the present context, this is obtained by first consolidating the agent’s budget constraints into the intertemporal form

$$\begin{aligned} & \frac{[1 + r_{t+1}(1 - \tau_{t+1}^k)](1 + \tau_t^c)}{(1 + \tau_{t+1}^c)} C_{1,t} + C_{2,t+1} \\ &= \frac{w_{2,t+1} l_{2,t+1} [1 - \tau_{2,t+1}^w]}{(1 + \tau_{t+1}^c)} + \frac{w_{1,t} l_{1,t} [1 - \tau_{1,t}^w] [1 + r_{t+1}(1 - \tau_{t+1}^k)]}{(1 + \tau_{t+1}^c)} \end{aligned} \quad (9)$$

and substituting the consumer optimality conditions (3a) – (3c), to yield

$$(C_{1,t} U_{C_{1,t}}^1 + l_{1,t} U_{l_{1,t}}^1) + \beta (C_{2,t+1} U_{C_{2,t+1}}^2 + l_{2,t+1} U_{l_{2,t+1}}^2) = 0 \quad (10)$$

This is the implementability constraint, i.e. a constraint among the allocations determined by the tax rates confronting the agent and is applied to each generation.<sup>16</sup>

We thus define the pseudo-welfare function to include the constraint

$$\begin{aligned} W_t' \equiv & U^1(C_{1,t}, l_{1,t}, G_{1,t}) + \beta U^2(C_{2,t+1}, l_{2,t+1}, G_{2,t+1}) \\ & + \lambda_t [(C_{1,t} U_{C_{1,t}}^1 + l_{1,t} U_{l_{1,t}}^1) + \beta (C_{2,t+1} U_{C_{2,t+1}}^2 + l_{2,t+1} U_{l_{2,t+1}}^2)] \end{aligned} \quad (11)$$

with the Lagrange multiplier  $\lambda_t > 0$ , when the constraint is binding and distortionary taxation is being employed. Written in this way,  $\lambda_t$  measures the marginal benefits to government revenue

<sup>15</sup> Early discussions of this are provided by Atkinson and Stiglitz (1980) and Lucas and Stokey (1983). More recently it has been developed further by Chari and Kehoe (1999), Atkeson, Chari, and Kehoe (1999) and Erosa and Gervais (2002). An excellent exposition is provided by Erosa and Gervais (2001).

<sup>16</sup> We should point out that this specific specification of the implementability condition is dependent upon there being age-dependent wage tax rates. If that were not the case, so that  $w_{1,t} = w_{2,t} = w_t$ , then (2a) and (2b) would become  $-U_{l_{1,t}}^1 / U_{C_{1,t}}^1 = w_{1,t} (1 - \tau_t^w) / (1 + \tau_t^c)$ ,  $-U_{l_{2,t+1}}^2 / U_{C_{2,t+1}}^2 = w_{2,t+1} (1 - \tau_{t+1}^w) / (1 + \tau_{t+1}^c)$  and would imply the additional constraint  $-U_{l_{1,t+1}}^1 / w_{1,t+1} U_{C_{1,t+1}}^1 = (1 - \tau_{t+1}^w) / (1 + \tau_{t+1}^c) = -U_{l_{2,t+1}}^2 / w_{2,t+1} U_{C_{2,t+1}}^2$

from an extra unit of distortionary taxes levied at time  $t$ , expressed in utility units.<sup>17</sup>

The Ramsey problem can thus be restated in terms of the following central planning problem

$$\text{Max } \delta^{-1}U_0^2 + \sum_{t=0}^{\infty} \delta^t W_t' \quad (7')$$

subject to

$$K_{t+1} - K_t = (1 - g_{1,t} - g_{2,t})F(K_t, l_{1,t}, l_{2,t}) - C_{1,t} - C_{2,t} \quad (6')$$

The central planner's problem is to maximize the discounted pseudo social welfare function, (7'), subject to the economy-wide resource constraint. Because the implementability constraint incorporates agents' intertemporal budget constraints, and since the problem explicitly incorporates the aggregate resource constraint, it follows that the government budget constraint is also automatically met.<sup>18</sup>

In describing the optimality conditions, the following quantities are critical:

$$H^{C_{i,t}} = \frac{C_{i,t}U^i_{C_{i,t}C_{i,t}} + l_{i,t}U^i_{l_{i,t}C_{i,t}}}{U^i_{C_{i,t}}}, \quad H^{l_{i,t}} = \frac{l_{i,t}U^i_{l_{i,t}l_{i,t}} + C_{i,t}U^i_{C_{i,t}l_{i,t}}}{U^i_{l_{i,t}}}, \quad H^{G_{i,t}} = \frac{C_{i,t}U^i_{C_{i,t}G_{i,t}} + l_{i,t}U^i_{l_{i,t}G_{i,t}}}{U^i_{G_{i,t}}} \quad i = 1, 2$$

Atkeson, Chari, and Kehoe (1999) characterize these types of measures as “general equilibrium elasticities” since they capture the distortions for setting income tax rates in general equilibrium. By incorporating the choice of government expenditure, we introduce the term  $H^{G_{i,t}}$ , which was not relevant in previous work in which government spending played no essential role.

#### 4.1 Government spending shares set arbitrarily

It is expositionally useful to consider first the optimality conditions for given government spending shares,  $g_{1,t}, g_{2,t}$ , and then to choose government spending optimally as well. Optimizing with respect to  $C_{1,t}, C_{2,t}, l_{1,t}, l_{2,t}, K_t$  yields the conditions

<sup>17</sup> In the infinite horizon representative agent model, the implementability constraint can be viewed as either the intertemporal budget constraint for the household or for the government; see Alvarez, Kehoe, and Neumeyer (2004). In the present framework with the household having a finite horizon and the government an infinite horizon this relationship is not so straightforward.

<sup>18</sup> This is demonstrated by Atkeson, Chari, and Kehoe (1999) and the same procedure can be applied here.

$$[1 + \lambda_t(1 + H^{C_{1,t}})]U_{C_{1,t}}^1 = \varphi_t \quad (12a)$$

$$\frac{\beta}{\delta}[1 + \lambda_{t-1}(1 + H^{C_{2,t}})]U_{C_{2,t}}^2 = \varphi_t \quad (12b)$$

$$\begin{aligned} [1 + \lambda_t(1 + H^{l_{1,t}})]U_{l_{1,t}}^1 + [1 + \lambda_t H^{G_{1,t}}]g_{1,t}F_{l_{1,t}}U_{G_{1,t}}^1 \\ + \frac{\beta}{\delta}g_{2,t}F_{l_{1,t}}U_{G_{2,t}}^2 [1 + \lambda_{t-1}H^{G_{2,t}}] = -\varphi_t(1 - g_{1,t} - g_{2,t})F_{l_{1,t}} \end{aligned} \quad (12c)$$

$$\begin{aligned} [1 + \lambda_{t-1}(1 + H^{l_{2,t}})]U_{l_{2,t}}^2 + [1 + \lambda_t H^{G_{1,t}}]g_{1,t}F_{l_{2,t}}U_{G_{1,t}}^1 \\ + \frac{\beta}{\delta}g_{2,t}F_{l_{2,t}}U_{G_{2,t}}^2 [1 + \lambda_{t-1}H^{G_{2,t}}] = -\varphi_{t-1}(1 - g_{1,t} - g_{2,t})F_{l_{2,t}} \end{aligned} \quad (12d)$$

$$U_{G_{1,t}}^1 g_{1,t}F_{K,t}[1 + \lambda_t H^{G_{1,t}}] + \frac{\beta}{\delta}U_{G_{2,t}}^2 g_{2,t}F_{K,t}[1 + \lambda_{t-1}H^{G_{2,t}}] + \varphi_t [(1 - g_{1,t} - g_{2,t})F_{K,t} + 1] = \left(\frac{\varphi_{t-1}}{\delta}\right) \quad (12e)$$

These five equations are analogous to equations (8a) – (8e), characterizing the unrestricted first-best optimum. They are standard efficiency conditions, which equate the social marginal costs to the social marginal benefits of consumption, labor and capital. There are two key differences from the previous optimality conditions, set out in (8). First, the marginal utilities must take account of the implementability constraint, (10), with the extent to which this is binding being reflected by whether  $\lambda_t > 0$  or not. The second is the fact that with government expenditures being specified as shares of growing output, this generates potential externalities.

To understand the intuition of these conditions consider first equation (12a), rewritten as

$$U_{C_{1,t}}^1 + \lambda_t \frac{\partial}{\partial C_{1,t}} (C_{1,t}U_{C_{1,t}}^1 + l_{1,t}U_{l_{1,t}}^1) = \varphi_t \quad (12a')$$

The first term measures the direct marginal utility to the young from an extra unit of consumption. To the extent that the government is imposing distortionary taxes, the second term measures the additional social benefits of the contribution to the current government surplus, expressed in terms of utility units. This term also reflects the effect of the additional consumption on the marginal utility derived from the existing consumption and leisure. A similar interpretation applies to (12b).

Next, (12c) equates the net marginal utility benefits of supplying young labor to its net

marginal resource costs. We can break down the former into three components. First, to the extent that agents dislike working, a unit increase in young labor supply directly decreases the utility of the young cohort by  $[1+\lambda_t(1+H^{l_t})]U_{l_t}^1$ . Second, by increasing output by  $F_{l_t,t}$ , the share of output spent by the government on the young cohort increases by  $g_{1,t}F_{l_t,t}$ , resulting in a utility gain to the young of  $[1+\lambda_t H^{G_{1,t}}]U_{G_{1,t}}^1 g_{1,t}F_{l_t,t}$ . This is the *within-cohort* externality. Third, at the same time, the spending on the older cohort increases by  $g_{2,t}F_{l_t,t}$ , raising their marginal utility by  $\frac{\beta}{\delta}U_{G_{2,t}}^2 [1+\lambda_{t-1}H^{G_{2,t}}]g_{2,t}F_{l_t,t}$ , this being the *cross-cohort* externality. In equilibrium these net utility gains must equal the resource costs measured in utility units. With government spending tied to output, this is given by  $-\varphi_t(1-g_{1,t}-g_{2,t})F_{l_t,t}$ . An analogous interpretation applies to (12d) and (12e), the optimality conditions with respect to old labor and capital.

The key point is that when the government ties spending to output, an increase in any productive factor generates both indirect benefits and costs. The benefits are the extra utility gains measured by the within-cohort and cross-cohort externalities, while the cost is in the form of the induced government claim on output.

## 4.2 Government spending shares set optimally

The optimality conditions (12a) – (12e) are derived under the assumption that both  $g_{1,t}, g_{2,t}$  are set arbitrarily. Choosing them optimally yields the two additional conditions:

$$U_{G_{1,t}}^1 [1+\lambda_t H^{G_{1,t}}] = \varphi_t \quad (13a)$$

$$\frac{\beta}{\delta} U_{G_{2,t}}^2 [1+\lambda_{t-1} H^{G_{2,t}}] = \varphi_t \quad (13b)$$

Conditions (13a) and (13b) show that marginal utility of government spending are equal to the shadow value of wealth for both young and the old. Combining these optimality conditions with (12a) and (12b), we obtain the following relationship.

$$[1+\lambda_t(1+H^{C_{1,t}})]U_{C_{1,t}}^1 = U_{G_{1,t}}^1 [1+\lambda_t H^{G_{1,t}}] = \frac{\beta}{\delta} U_{G_{2,t}}^2 [1+\lambda_{t-1} H^{G_{2,t}}] = \frac{\beta}{\delta} [1+\lambda_{t-1}(1+H^{C_{2,t}})]U_{C_{2,t}}^2 = \varphi_t \quad (14)$$

This relationship links the marginal benefits of consumption to the marginal benefits of government

spending for both young and old, identifying the socially optimal level of cohort-specific government spending. We will use this relationship as a benchmark to interpret the consequences of setting  $g_{1,t}$  and  $g_{2,t}$  non-optimally.

## 5. Steady-state

In principle, the macroeconomic equilibrium set out in equations (12) [and (13)], coupled with the resource constraint, (6'), and the implementability constraint, (10), describe the evolution of the economy over time. Since much of the interest lies in the long-term fiscal structure – i.e. the composition of government expenditure and its financing – we shall focus our attention on the steady state. We shall present this in two stages, first for arbitrarily set government expenditure shares, and then for optimally set government expenditures. In each case, we shall set out the optimal quantity allocations followed by the restrictions on the tax rates necessary to achieve them.

### 5.1 Arbitrarily set government spending

We shall denote the steady states to this Ramsey problem by bars. Thus we obtain

#### 5.1.1 Optimal quantity allocations

Private consumption

$$\left[1 + \bar{\lambda}(1 + H^{C_1})\right] U_c^1(\bar{C}_1, \bar{l}_1, G_1) = \bar{\varphi} \quad (15a)$$

$$\left[1 + \bar{\lambda}(1 + H^{C_2})\right] \frac{\beta}{\delta} U_c^2(\bar{C}_2, \bar{l}_2, G_2) = \bar{\varphi} \quad (15b)$$

Private labor supply

$$g_1 F_l \left\{ \overbrace{\left[1 + \bar{\lambda} H^{G_1}\right] U_{G_1}^1 - \left[1 + \bar{\lambda}(1 + H^{C_1})\right] U_{C_1}^1}^{\text{deviation from optimal } g_1} \right\} + \frac{\beta}{\delta} g_2 F_l \left\{ \overbrace{\left[1 + \bar{\lambda} H^{G_2}\right] U_{G_2}^2 - \left[1 + \bar{\lambda}(1 + H^{C_2})\right] U_{C_2}^2}^{\text{deviation from optimal } g_2} \right\} + \bar{\varphi} F_l = -\left[1 + \bar{\lambda}(1 + H^h)\right] U_l^1 \quad (15c)$$

$$\begin{aligned}
g_1 F_{l_2} \left\{ \overbrace{[1 + \bar{\lambda} H^{G_1}] U_{G_1}^1 - [1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1}^{\text{deviation from optimal } g_1} \right\} + \frac{\beta}{\delta} g_2 F_{l_2} \left\{ \overbrace{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2 - [1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2}^{\text{deviation from optimal } g_2} \right\} \\
+ \bar{\varphi} F_{l_2} = -\frac{\beta}{\delta} [1 + \bar{\lambda} (1 + H^{l_2})] U_{l_2}^2 \quad (15d)
\end{aligned}$$

Private capital

$$\begin{aligned}
g_1 F_K \left\{ \overbrace{[1 + \bar{\lambda} H^{G_1}] U_{G_1}^1 - [1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1}^{\text{deviation from optimal } g_1} \right\} + \frac{\beta}{\delta} g_2 F_K \left\{ \overbrace{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2 - [1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2}^{\text{deviation from optimal } g_2} \right\} \\
+ \bar{\varphi} F_K = \bar{\varphi} \left( \frac{1 - \delta}{\delta} \right) \quad (15e)
\end{aligned}$$

Implementability constraint

$$\bar{\lambda} \cdot \left\{ [\bar{C}_1 U_c^1(\cdot) + \bar{l}_1 U_l^1(\cdot)] + \beta [\bar{C}_2 U_c^2(\cdot) + \bar{l}_2 U_l^2(\cdot)] \right\} = 0 \quad (15f)$$

Goods market clearance

$$F(\bar{K}, \bar{l}_1, \bar{l}_2) = \bar{C}_1 + \bar{C}_2 + G_1 + G_2 \quad (15g)$$

where  $G_1 = g_1 F(\bar{K}, \bar{l}_1, \bar{l}_2)$ ,  $G_2 = g_2 F(\bar{K}, \bar{l}_1, \bar{l}_2)$ . These 7 equations determine the optimal steady-state Ramsey allocations of:  $\bar{C}_1, \bar{C}_2, \bar{l}_1, \bar{l}_2, \bar{K}, \bar{\varphi}, \bar{\lambda}$ .

There are two points we wish to make about this equilibrium. First we have rewritten the optimality conditions with respect to the productive factors,  $l_1, l_2, K$ , [(12c) – (12e)] to highlight the externality generated by non-optimal government spending. Specifically, the optimality conditions with respect to  $l_1, l_2, K$  in (15c) – (15e) include the terms  $\{[1 + \bar{\lambda} H^{G_i}] U_{G_i}^i - [1 + \bar{\lambda} (1 + H^{C_i})] U_{C_i}^i\}$ . These reflect the deviations of the arbitrarily set  $g_i$  from their respective optimal steady-state values. If  $g_1$  and  $g_2$  are chosen optimally, then these terms all vanish and the marginal products of young and old labor are equal to the marginal rate of substitution between labor and consumption for the respective cohort.

The second point to note is that we have written the implementability constraint as a more

general Kuhn-Tucker condition.<sup>19</sup> As we have noted, the significance of this condition depends upon the set of fiscal instruments available to the government. In the event that it has sufficient instruments to replicate the first-best optimum of the unconstrained central planner,  $\bar{\lambda} = 0$  and the implementability constraint becomes irrelevant.

### 5.1.2 Fiscal constraints

These allocations must be consistent with the tax and financing constraints, which we may write as follows; see Appendix:

$$\frac{1 - \tau_1^w}{1 + \tau^c} = \frac{1 + \bar{\lambda}(1 + H^{C_1})}{1 + \bar{\lambda}(1 + H^h)} \left\{ 1 + g_1 \left( \frac{[1 + \bar{\lambda}H^{G_1}]U_{G_1}^1}{[1 + \bar{\lambda}(1 + H^{C_1})]U_{C_1}^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda}H^{G_2}]U_{G_2}^2}{[1 + \bar{\lambda}(1 + H^{C_2})]U_{C_2}^2} - 1 \right) \right\} \quad (16a)$$

$$\frac{1 - \tau_2^w}{1 + \tau^c} = \frac{[1 + \bar{\lambda}(1 + H^{C_2})]}{[1 + \bar{\lambda}(1 + H^{L_2})]} \left\{ 1 + g_1 \left( \frac{[1 + \bar{\lambda}H^{G_1}]U_G^1}{[1 + \bar{\lambda}(1 + H^{C_1})]U_C^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda}H^{G_2}]U_{G_2}^2}{[1 + \bar{\lambda}(1 + H^{C_2})]U_{C_2}^2} - 1 \right) \right\} \quad (16b)$$

$$\begin{aligned} & \frac{[1 + \bar{\lambda}(1 + H^{C_1})]}{[1 + \bar{\lambda}(1 + H^{C_2})]} [1 + (1 - \tau^k)F_K] \\ &= [F_K + 1] + g_1 F_K \left\{ \frac{U_G^1 [1 + \bar{\lambda}H^{G_1}]}{[1 + \bar{\lambda}(1 + H^{C_1})]U_{C_1}^1} - 1 \right\} + g_2 F_K \left\{ \frac{U_{G_2}^2 [1 + \bar{\lambda}H^{G_2}]}{[1 + \bar{\lambda}(1 + H^{C_2})]U_{C_2}^2} - 1 \right\} \end{aligned} \quad (16c)$$

In addition, the tax rates must be consistent with the steady-state government budget constraint.

Invoking the equilibrium factor returns this can be written as:

$$\begin{aligned} & (g_1 + g_2)F(\bar{K}, \bar{l}_1, \bar{l}_2) + F_K(\bar{K}, \bar{l}_1, \bar{l}_2)\bar{B} \\ &= \tau_1^w F_{l_1}(\bar{K}, \bar{l}_1, \bar{l}_2)\bar{l}_1 + \tau_2^w F_{l_2}(\bar{K}, \bar{l}_1, \bar{l}_2)\bar{l}_2 + \tau^c (\bar{C}_1 + \bar{C}_2) + \tau^k F_K(\bar{K}, \bar{l}_1, \bar{l}_2)(\bar{K} + \bar{B}) \end{aligned} \quad (16d)$$

Given the factor allocations, and the arbitrarily set government expenditure shares,  $g_1$  and  $g_2$ , equations (16a) and (16b) determine constraints between the taxes on labor income,

<sup>19</sup> In order to interpret the Lagrange multiplier associated with the implementability constraint appropriately, one should introduce the constraint as an inequality. Since the constraint imposes conditions on the first derivatives of the utility function, the second-best optimal tax problem that it encompasses may be associated with non-convexities. A careful treatment of these issues is provided by de la Croix and Michel (2002). See also Persson, Persson, and Svensson (2006) who formulate the implementability constraint as an inequality constraint.

$\tau_1^w, \tau_2^w$ , and on consumption,  $\tau^c$  that must hold, while (16c) determines the corresponding second-best optimal tax on capital,  $\tau^k$ . Given  $\tau_1^w, \tau_2^w$  and  $\tau^k$ , the budget constraint (16d) determines the combination of debt and  $\tau^c$  that will finance the government expenditures. The main point to observe is that the externality created by the non-optimal choices of  $g_1, g_2$ , is an important component of the optimal tax policy. To understand this further, it is convenient to focus first on the case where the government chooses its expenditure policies optimally.

## 5.2 Optimally set government expenditure

When government expenditure is set optimally the steady-state equilibrium quantities (denoted by  $\hat{\cdot}$ ) are:

$$\left[1 + \hat{\lambda}(1 + H^{c_1})\right] U_c^1(\hat{C}_1, \hat{l}_1, \hat{G}_1) = \hat{\phi} \quad (15a')$$

$$\left[1 + \hat{\lambda}(1 + H^{c_2})\right] \frac{\beta}{\delta} U_c^2(\hat{C}_2, \hat{l}_2, \hat{G}_2) = \hat{\phi} \quad (15b')$$

$$[1 + \hat{\lambda}(1 + H^l)] U_l^1 = -\hat{\phi} F_l \quad (15c')$$

$$\frac{\beta}{\delta} [1 + \hat{\lambda}(1 + H^l)] U_l^2 = -\hat{\phi} F_l \quad (15d')$$

$$F_k = \left(\frac{1 - \delta}{\delta}\right) \quad (15e')$$

$$\hat{\lambda} \cdot \left\{ [C_1 U_c^1(\cdot) + l_1 U_l^1(\cdot)] + \beta [C_2 U_c^2(\cdot) + l_2 U_l^2(\cdot)] \right\} = 0 \quad (15f')$$

$$F(\hat{K}, \hat{l}_1, \hat{l}_2) = \hat{C}_1 + \hat{C}_2 + \hat{G}_1 + \hat{G}_2 \quad (15g')$$

$$[1 + \hat{\lambda} H^{G_1}] U_G^1(\hat{C}_1, \hat{l}_1, \hat{G}_1) = \hat{\phi} \quad (15h)$$

$$[1 + \hat{\lambda} H^{G_2}] \frac{\beta}{\delta} U_G^2(\hat{C}_2, \hat{l}_2, \hat{G}_2) = \hat{\phi} \quad (15i)$$

The first issue we wish to check is whether or not the replication of equations (8a) – (8j) of the first-best optimum is possible, so that the implementability constraint does not apply. In other words, is it possible to choose tax rates consistent with the government budget constraint, such that

(i) the first-best optimal allocation is attained, and (ii)  $\hat{\lambda} = 0$  ?

Setting  $\hat{\lambda} = 0$  these equations do indeed coincide with the first-best optimum, so that the first-best optimal allocations are achieved. The question then arises as to whether this is feasible, given the available set of policy instruments. Invoking equations (15a'), (15b'), (15h) and (15i), with  $\hat{\lambda} = 0$ , (16a) – (16c) simplify drastically to:

$$\hat{\tau}_1^w = \hat{\tau}_2^w = -\hat{\tau}_c \equiv \zeta \quad (17a)$$

$$\hat{\tau}_k = 0 \quad (17b)$$

Since there is no distortion in the labor-consumption tradeoff, both cohorts should have their labor incomes taxed at a common rate, with the consumption tax (subsidy) being equal and opposite, while the tax on capital should be zero.<sup>20</sup> Substituting these optimal tax rates into (16d), the government budget constraint, requires

$$\hat{B}F_K(\hat{K}, \hat{l}_1, \hat{l}_2) + \hat{G}_1 + \hat{G}_2 = \zeta \left[ F_{l_1}(\hat{K}, \hat{l}_1, \hat{l}_2)\hat{l}_1 + F_{l_2}(\hat{K}, \hat{l}_1, \hat{l}_2)\hat{l}_2 - (\hat{C}_1 + \hat{C}_2) \right] \quad (17c)$$

which in general can be achieved. We may thus state

**Proposition 1:** As long as the government has at its disposal a consumption tax it can attain the first-best allocation of resources. This involves a zero tax on capital income, and taxing the labor income of cohorts uniformly. If the government has positive outstanding debt, and total labor income exceeds (is less than) total consumption expenditures the tax on labor income should be positive (negative), with consumption subsidized (taxed) at the same rate.

The more interesting case arises when a consumption tax is unavailable. In that case, the implementability constraint applies,  $\hat{\lambda} > 0$ , and the implied optimal income tax rates are:

$$\hat{\tau}_1^w = \frac{\hat{\lambda}(H^h - H^{C_1})}{1 + \hat{\lambda}(1 + H^h)} \quad (16a')$$

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<sup>20</sup> The structure of the optimal tax policy as set out in (17a) and (17b) in this life-cycle model is similar to the steady-state structure obtained by Turnovsky (2000) for the infinitely-lived representative agent economy.

$$\hat{\tau}_2^w = \frac{\hat{\lambda}(H^{l_2} - H^{c_2})}{1 + \hat{\lambda}(1 + H^{l_2})} \quad (16b')$$

$$\hat{\tau}^k = \frac{\hat{\lambda}(H^{c_1} - H^{c_2})}{[1 + \hat{\lambda}(1 + H^{c_1})]1 - \delta} \quad (16c')$$

Given the optimal tax rates as reported in (16a') – (16c'), the government budget constraint determines the implied equilibrium stock of government bonds,  $\hat{B}$ :

$$\hat{B}F_K(\hat{K}, \hat{l}_1, \hat{l}_2) + \hat{G}_1 + \hat{G}_2 = \hat{\tau}_1^w F_{l_1}(\hat{K}, \hat{l}_1, \hat{l}_2)\hat{l}_1 + \hat{\tau}_2^w F_{l_2}(\hat{K}, \hat{l}_1, \hat{l}_2)\hat{l}_2 + \hat{\tau}^k F_K(\hat{K}, \hat{l}_1, \hat{l}_2)(\hat{K} + \hat{B}) \quad (16d)$$

The expressions for the optimal tax rates are similar to those obtained by Erosa and Gervais (2001). In general, as long as the generalized elasticities,  $H^{c_i}$ ,  $H^{l_i}$ , vary across the life cycle, they call for the unequal taxation of the labor income of each cohort, and for a non-zero long-run tax on capital. Moreover, the impact of government spending on the optimal tax structure will depend upon the effect of government spending on the generalized elasticities, as well as on  $\hat{\lambda}$ .

Further insight is obtained by considering the special case where the agent's two period utility function is of the homogeneous-separable form:

$$W \equiv U^1(C_1, 1 - l_1)V^1(G_1) + \beta[U^2(C_2, 1 - l_2)V^2(G_2)] \quad (18)$$

and  $U^i$  is homogeneous of degree one in consumption and leisure. From the definitions of the generalized elasticities we can show that homogeneity implies

$$H^{l_i} - H^{c_i} = \frac{U_{l_i}^i U^i}{U_{C_i}^i U^i C_i} > 0 \quad i = 1, 2 \quad (19a)$$

$$H^{c_1} - H^{c_2} = \frac{U_{C_1}^1 C_1}{U_{C_1}^1 (1 - l_1)} - \frac{U_{C_2}^2 C_2}{U_{C_2}^2 (1 - l_2)} \quad (19b)$$

Thus we can state:

**Proposition 2:** Suppose that government expenditure on each cohort is chosen optimally and that utility is of the form (18). Then, in general:

- (i) labor income of the different cohorts should be taxed positively and

differentially, and,

- (ii) the tax on capital income should be non-zero.

The implications of Proposition 2 differ sharply from those of Proposition 1. To understand the intuition underlying this contrast it is helpful to rewrite the relationships (16a) – (16c) characterizing the optimal tax rates in the form

$$1 - \tau_i \equiv \frac{1 - \tau_i^w}{1 + \tau^c} = \frac{1 + \hat{\lambda}(1 + H^{C_i})}{1 + \hat{\lambda}(1 + H^{L_i})} = - \frac{\hat{U}_{l_i}^i / \hat{U}_{c_i}^i}{\hat{F}_{l_i}} \quad i = 1, 2 \quad (16a'')$$

$$\frac{[1 + \hat{\lambda}(1 + H^{C_1})]}{[1 + \hat{\lambda}(1 + H^{C_2})]} \left[ \frac{1}{\delta} - \tau^k \left( \frac{1 - \delta}{\delta} \right) \right] = \frac{1}{\delta} \quad (16c'')$$

where  $\tau_i \equiv (\tau_i^w + \tau_c) / (1 + \tau_c)$  is the effective tax on cohort  $i$ 's labor income [Prescott, 2004]. In the situation where the availability of the consumption tax enables the first-best optimum to be attained, so that  $\hat{\lambda} = 0$ , then in the absence of any distortions in the labor market the effective tax on labor income should be zero, i.e.  $\tau_i = 0$ . With the consumption tax being uniform across cohorts, this requires the labor income tax to be uniform as well.<sup>21</sup> Moreover, with the capital market being distortion-free, the capital income tax should be zero. However, if the consumption tax is unavailable so that the implementability constraint is therefore in effect ( $\hat{\lambda} > 0$ ), agents at different stages of their life-cycle will experience different degrees of distortions, requiring a differential effective tax on labor income to correct. With no consumption tax this requires a differential tax on labor income.

These expressions sharpen further if we assume that  $U^i$  is of the widely employed constant elasticity form,  $U^i = C_i^\alpha (1 - l_i)^{1 - \alpha}$ , where utility is uniform across cohorts. In this case we find

$$H^{L_i} - H^{C_i} = \frac{1}{1 - l_i} > 0 \quad (19a')$$

$$H^{C_1} - H^{C_2} = \frac{(1 - \alpha)(l_2 - l_1)}{(1 - l_1)(1 - l_2)} \quad (19b')$$

---

<sup>21</sup> If the consumption tax were non-uniform across cohorts, then the labor income tax would need to be non-uniform as well, exactly offsetting the differentials in the consumption tax.

In particular from (19a')-(19b'), in conjunction with (16a')-(16c') we infer that

$$\tau_1^w > \tau_2^w > 0, \text{ and } \tau^k < 0 \text{ if and only if } l_1 > l_2 \quad (20)$$

Thus the tax on labor income of the young exceeds the tax on labor income of the old, while the tax on capital income is negative if and only if the fraction of time devoted to work of the young exceeds that of the old. In this case we may modify Proposition 2 to:

**Proposition 2'**: If preferences are of the constant elasticity utility form, then

- (i) the labor income of the cohort spending the longer work time should be taxed more heavily, and
- (ii) the tax on capital should be negative if and only if the young devote a greater fraction of their time to work than do the old.

These results can be given the following straightforward intuition. If, for example,  $l_1 > l_2$ , so that the young work more than the old, the old enjoy more leisure. Since leisure and consumption are complementary in utility, the old will derive more benefits from consumption than do the young. As a consequence, the government should encourage agents to postpone some of their consumption from when they are young to when they are old. To encourage savings so as to achieve this they should impose a negative tax on the capital income earned by the old, as well as taxing the labor income of the old at a lower rate. The opposite applies if  $l_1 < l_2$ .

Being a general equilibrium model, the steady-state values of  $l_1$  and  $l_2$  are endogenous. Given uniform preferences across the two cohorts, the relative time worked by the two cohorts will depend upon their relative productivities. Assuming a linearly, homogeneous Cobb-Douglas production function,  $F = Al_1^{\theta_1}l_2^{\theta_2}K^{1-\theta_1-\theta_2}$ , we show in the Appendix that  $l_1 > l_2$  if and only if  $\delta\theta_1 > \beta\theta_2$ . Thus we immediately infer:

**Proposition 3:** For the Cobb-Douglas production function, the optimal tax structure set out in Proposition 2' will occur if and only if the ratio of the cohorts' productivities exceeds the ratio of the agent's time preference to the central planner's intergenerational discount factor, i.e. if and only if  $\theta_1/\theta_2 > \beta/\delta$ .

Proposition 3 thus implies that in the case that the two cohorts are equally productive, i.e.  $\theta_1 = \theta_2$ , then  $\tau_k < 0$  if and only if  $\delta > \beta$ . That is, capital is subsidized if and only if the central planner cares more about future generations than the young cohort does about the old.

### 5.3 Optimal tax structure with non-optimally set government expenditure

As long as government spending is set optimally, its impact on the optimal tax structure is relatively minor. For example, for the class of utility function represented in Proposition 2' its role in determining whether the tax on capital income should be positive or negative operates only through its impact on the relative labor supply,  $l_1 - l_2$ .<sup>22</sup> However, the role of government spending becomes much more important when it is set non-optimally. To see this, we return to (16a) – (16c) to reinterpret the impact of non-optimal government spending on the optimal tax structure.

As a benchmark, let us assume that  $H^{C_1} = H^{C_2} \equiv H^C$ ;  $H^{l_1} = H^{l_2} \equiv H^l$ , so that the generalized consumption and labor elasticities remain constant over the life cycle.<sup>23</sup> In that case, if government spending is set optimally, labor income of the two cohorts should be taxed uniformly at the rate

$$\hat{\tau}_1^w = \hat{\tau}_2^w \equiv \hat{\tau}^w = \frac{\hat{\lambda}(H^l - H^C)}{1 + \hat{\lambda}(1 + H^l)} \quad (21a)$$

while the optimal tax on capital,  $\hat{\tau}^k = 0$ ; see (16c').

Suppose now that the government sets its expenditure rates non-optimally. In this case, (16a)-(16c), together with (21a) [and  $\tau_c = 0$ ] imply

$$\frac{1 - \bar{\tau}_1^w}{1 - \hat{\tau}^w} = \left\{ 1 + g_1 \left( \frac{[1 + \bar{\lambda} H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} - 1 \right) \right\} \quad (22a)$$

$$\frac{1 - \bar{\tau}_2^w}{1 - \hat{\tau}^w} = \left\{ 1 + g_1 \left( \frac{[1 + \bar{\lambda} H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} - 1 \right) \right\} \quad (22b)$$

$$\bar{\tau}^k = g_1 \left\{ 1 - \frac{U_G^1 [1 + \bar{\lambda} H^{G_1}]}{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1} \right\} + g_2 \left\{ 1 - \frac{U_G^2 [1 + \bar{\lambda} H^{G_2}]}{[1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} \right\} \quad (22c)$$

<sup>22</sup> The effects of the government expenditure on the *levels* of the tax rates will also depend upon its effect on  $\hat{\lambda}$ .

<sup>23</sup> A simple example of a utility function having this property is  $U^i = aC_i^\alpha - bl_i^\beta + V(G_i)$ ,  $a, b > 0$ ,  $0 < \alpha < 1$ ,  $\beta > 1$ .

From these conditions we see that if the government spends too little on either cohort, so that  $[1 + \bar{\lambda} H^{G_i}] U_{G_i}^1 > \varphi$ , then  $\bar{\tau}_i^w < \hat{\tau}^w$ ,  $i = 1, 2$ , and  $\bar{\tau}_k < 0$ . In this case, the tax on labor income is set below that of the first-best equilibrium and the optimal tax on capital is negative. If the expenditure on either group is excessive, the opposite holds; the tax on labor income should be set above the optimum and the optimal tax on capital is positive. By choosing non-optimal expenditure shares the government creates distortions at intra- and intertemporal margins. Therefore, taxes are required to correct these distortions. The result on capital income tax shows that optimal distortionary taxes produce a net distortion in the direction opposite to the intertemporal bias in benefits provided by the government.

One further case of interest arises if the overall government spending share is optimal, but its allocation across the cohorts is non-optimal. In that case the optimality conditions (13a) and (13b) which apply to the independent optimal choice of  $g_1, g_2$ , no longer hold. Instead, letting  $g_1 = \nu g$ ,  $g_2 = (1 - \nu)g$ , and optimizing over the total government expenditure share,  $g$ , for an arbitrarily given allocation  $\nu$ , we obtain the optimality condition

$$\nu g U_{G_{1,t}}^1 [1 + \lambda_t H^{G_{1,t}}] + (1 - \nu) g \frac{\beta}{\delta} U_{G_{2,t}}^2 [1 + \lambda_{t-1} H^{G_{2,t}}] = g \varphi_t \quad (23)$$

In steady-state equilibrium this converges to

$$g_1 U_{G_{1,t}}^1 [1 + \lambda H^{G_1}] + g_2 \frac{\beta}{\delta} U_{G_2}^2 [1 + \lambda H^{G_2}] = g \varphi$$

which we can rewrite in the form

$$g_1 \left( U_{G_{1,t}}^1 [1 + \lambda H^{G_1}] - \varphi \right) + g_2 \left( \frac{\beta}{\delta} U_{G_2}^2 [1 + \lambda H^{G_2}] - \varphi \right) = 0$$

Dividing by  $\varphi$  and using the optimality conditions (15a) and (15b), we see that (22a) – (22c) reduce to  $\tau_1^w = \tau_2^w = \hat{\tau}^w$ ,  $\tau^k = 0$ . In other words, the optimal tax structure of Proposition 2 continues to hold as long as *total* government spending is optimal, even if it is non-optimally allocated across the two cohorts.<sup>24</sup> We may summarize these results in:

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<sup>24</sup> This result is analogous to one obtained by Turnovsky (2000) in the infinite-lived representative agent framework, where the government spends fixed shares of output on a consumption good and on a productive input. There he showed

**Proposition 4:** If the government allocates too much (insufficient) output to either cohort, then the labor income of both cohorts should be taxed at above (below) the optimal rate and the tax on capital income should be positive (negative). In the case that overall fraction of government spending is optimal, but is incorrectly allocated to cohorts, the first best optimal tax structure applies.

The intuition for the last statement in Proposition 4 is as follows. If the government overspends on the young say, the optimal tax implies that the young cohort is undertaxed relative to what they should be. However, since the overspending on the young is just matched by underspending on the old, when the young become old, they receive a reduction in benefits, for which are they relatively overtaxed. With the overall government spending being optimal, these two effects exactly cancel in present value terms, so that overall welfare is optimized.

## 6. Conclusion

This paper has studied optimal capital and labor income taxes in a framework of the optimal provision of public goods when benefits are age dependent. A two-period overlapping generations model with endogenous labor supply in both periods has been employed to derive the optimal public spending on each cohort together with the optimal Ramsey taxes.

As long as the government has at its disposal a consumption tax, it can attain the first-best allocation of resources. This involves the uniform taxation of the cohorts' labor income and a zero tax on capital income. With no consumption tax and if government spending is chosen optimally, labor income should in general be taxed non-uniformly across cohorts and the capital income tax should be non zero. Deviations of the public goods from their respective optimal level creates distortions at the intra- and intertemporal margins and affects the labor supply and capital accumulation decisions of both young and old, providing a further reason to tax (or subsidize) capital income.

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that the optimal tax on capital depends upon the deviation of the *aggregate* share of government spending from its optimum; the composition of government expenditure is irrelevant to the optimal tax structure, just as it is here.

## Appendix

### A. Derivation of Equations (16)

We begin by rewriting equation (15c) in the form

$$g_1 F_l \left( [1 + \lambda H^{G_1}] U_{G_1}^1 - [1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1 \right) + \frac{\beta}{\delta} g_2 F_l \left( [1 + \bar{\lambda} H^{G_2}] U_{G_2}^2 - [1 + \bar{\lambda}(1 + H^{C_2})] U_{C_2}^2 \right) + \bar{\varphi} F_l = -[1 + \bar{\lambda}(1 + H^h)] U_l^1 \quad (\text{A.1})$$

Divide by  $\bar{\varphi}$ , while recalling (15a) and (15b) yields

$$g_1 F_l \left( \frac{[1 + \lambda H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 F_l \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda}(1 + H^{C_2})] U_{C_2}^2} - 1 \right) + F_l = - \frac{[1 + \bar{\lambda}(1 + H^h)] U_l^1}{[1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1} \quad (\text{A.2})$$

Substituting the optimality condition (3a), which in steady-state equilibrium is

$$-\frac{U_l^1}{U_{C_1}^1} = F_l \left[ \frac{1 - \tau_1^w}{1 + \tau^c} \right]$$

into (A.2), we obtain

$$g_1 \left( \frac{[1 + \lambda H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda}(1 + H^{C_2})] U_{C_2}^2} - 1 \right) + 1 = \frac{[1 + \bar{\lambda}(1 + H^h)]}{[1 + \bar{\lambda}(1 + H^{C_1})]} \left( \frac{1 - \tau_1^w}{1 - \tau^c} \right)$$

from which we immediately obtain the relationship

$$\left( \frac{1 - \tau_1^w}{1 - \tau^c} \right) = \frac{[1 + \bar{\lambda}(1 + H^{C_1})]}{[1 + \bar{\lambda}(1 + H^h)]} \left\{ 1 + g_1 \left( \frac{[1 + \lambda H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda}(1 + H^{C_2})] U_{C_2}^2} - 1 \right) \right\} \quad (\text{A.3})$$

which is (16a) of the text. Equation (16b) is obtained similarly.

To derive (16c) we begin with (15e), expressed as

$$g_1 F_K \left( [1 + \bar{\lambda} H^{G_1}] U_{G_1}^1 - [1 + \bar{\lambda}(1 + H^{C_1})] U_{C_1}^1 \right) + \frac{\beta}{\delta} g_2 F_K \left( [1 + \bar{\lambda} H^{G_2}] U_{G_2}^2 - [1 + \bar{\lambda}(1 + H^{C_2})] U_{C_2}^2 \right) + \bar{\varphi} F_K = \bar{\varphi} \left( \frac{1 - \delta}{\delta} \right) \quad (\text{A.4})$$

Dividing this equation by  $\bar{\varphi}$  yields

$$g_1 F_K \left( \frac{[1 + \bar{\lambda} H^{G_1}] U_{G_1}^1}{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1} - 1 \right) + g_2 F_K \left( \frac{[1 + \bar{\lambda} H^{G_2}] U_{G_2}^2}{[1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} - 1 \right) + F_K + 1 = \frac{1}{\delta} \quad (\text{A.5})$$

Also, dividing (15b) by (15a) and combining with the optimality condition (3c), which in steady-state equilibrium is

$$\frac{U_{C_1}^1}{U_{C_2}^2} = \beta [1 + F_K (1 - \tau_{t+1}^k)]$$

yields

$$\frac{1}{\delta} = \frac{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1}{\beta [1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} = \frac{[1 + \bar{\lambda} (1 + H^{C_1})]}{[1 + \bar{\lambda} (1 + H^{C_2})]} [1 + F_K (1 - \tau_{t+1}^k)] \quad (\text{A.6})$$

Finally, equating (A.5) and (A.6) yields

$$\begin{aligned} & \frac{[1 + \bar{\lambda} (1 + H^{C_1})]}{[1 + \bar{\lambda} (1 + H^{C_2})]} [1 + (1 - \tau^k) F_K] \\ &= [F_K + 1] + g_1 F_K \left\{ \frac{U_G^1 [1 + \bar{\lambda} H^{G_1}]}{[1 + \bar{\lambda} (1 + H^{C_1})] U_{C_1}^1} - 1 \right\} + g_2 F_K \left\{ \frac{U_{G_2}^2 [1 + \bar{\lambda} H^{G_2}]}{[1 + \bar{\lambda} (1 + H^{C_2})] U_{C_2}^2} - 1 \right\} \end{aligned} \quad (\text{A.7})$$

which is (16c) of the text.

## B. Derivation of Proposition 3

Suppose that the production function is of the Cobb-Douglas form

$$F = A l_1^{\theta_1} l_2^{\theta_2} K^{1 - \theta_1 - \theta_2}$$

so that  $F_{l_1} = \theta_1 F / l_1$ ,  $F_{l_2} = \theta_2 F / l_2$ . Dividing the optimality condition (15c') by (15d') yields

$$\frac{[1 + \hat{\lambda} (1 + H^{l_1})] U_{l_1}^1}{[1 + \hat{\lambda} (1 + H^{l_2})] U_{l_2}^2} = \frac{\delta}{\beta} \frac{F_{l_1}}{F_{l_2}} \quad (\text{B.1})$$

which for the constant elasticity utility function and Cobb-Douglas production functions becomes

$$\frac{\left[1 + \hat{\lambda} \left(1 + \frac{\alpha}{1-l_1}\right)\right] (1-l_1)^{-\alpha}}{\left[1 + \hat{\lambda} \left(1 + \frac{\alpha}{1-l_2}\right)\right] (1-l_2)^{-\alpha}} = \frac{\delta \theta_1 l_2}{\beta \theta_2 l_1} \quad (\text{B.2})$$

which can be rewritten as

$$\frac{(1 + \hat{\lambda})(1-l_1)(1-l_2)^{1+\alpha} l_1 + \hat{\lambda} \alpha (1-l_2)^{1+\alpha} l_1}{(1 + \hat{\lambda})(1-l_2)(1-l_1)^{\alpha+1} l_2 + \hat{\lambda} \alpha (1-l_1)^{\alpha+1} l_2} = \frac{\delta \theta_1}{\beta \theta_2} \quad (\text{B.3})$$

Suppose  $\frac{\theta_1}{\theta_2} > \frac{\beta}{\delta}$ . Then

$$(1 + \hat{\lambda})(1-l_1)(1-l_2)^{1+\alpha} \left[ l_1(1-l_2)^\alpha - l_2(1-l_1)^\alpha \right] + \hat{\lambda} \alpha \left[ l_1(1-l_2)^{\alpha+1} - l_2(1-l_1)^{\alpha+1} \right] > 0 \quad (\text{B.4})$$

To show this implies  $l_1 > l_2$ , assume the contrary. It then follows that  $l_1(1-l_2)^\alpha < l_2(1-l_1)^\alpha$ ,  $l_1(1-l_2)^{\alpha+1} < l_2(1-l_1)^{\alpha+1}$  contradicting (B.4), so that  $l_1 > l_2$ . Likewise  $\frac{\theta_1}{\theta_2} < \frac{\beta}{\delta}$  implies  $l_2 > l_1$ . Moreover, the argument can be reversed, implying  $l_1 > l_2$  if and only if  $\frac{\theta_1}{\theta_2} > \frac{\beta}{\delta}$ .

## References

- ALVAREZ, F., P.J. KEHOE, and P. NEUMEYER (2004) The time consistency of optimal monetary and fiscal policies, *Econometrica* **72**, 541-567.
- ATKESON, A., V.V. CHARI, and P. KEHOE (1999) Taxing capital income: A bad idea, *Federal Reserve Bank of Minneapolis Quarterly Review* **23/3**, 3-17.
- ATKINSON, A.K. and J.E. STIGLITZ (1980) *Lectures on Public Economics*. New-York, McGraw-Hill.
- AUERBACH, A.J., J. GOKHALE, and L.J. KOTLIKOFF (1991) Generational accounts: A meaningful alternative to deficit accounting, *Tax Policy and the Economy* **5**, 55-110.
- BARRO, R.J. (1990) Government spending in a simple model of endogenous growth, *Journal of Political Economy* **98**, S103-125.
- CHAMLEY, C. (1986) Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* **54**, 607-622.
- CHARI, V.V. and P.J. KEHOE (1999) Optimal monetary and fiscal policy, in J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics Vol 1*. Amsterdam: Elsevier, 1671-1745.
- CORREIA, I., (1996) Should capital income be taxed in the steady state? *Journal of Public Economics* **60**, 147-151.
- CREMER, H., PESTIEAU, and P. ROCHET, J.C. (2003) Capital income taxation when inherited wealth is not observable, *Journal of Public Economics* **87**, 2475-2490.
- CRETTEZ B. and LE MAITRE P. (2002) Optimal date of retirement and population growth, *Journal of Population Economics* **15**, 737-755.
- CROIX, DE LA, D. and P. MICHEL (2002) *A Theory of Economic Growth*. Cambridge UK: Cambridge University Press.
- EROSA, A. and M. GERVAIS (2001) Optimal taxation in infinitely-lived agent and overlapping generations models: A review, *Federal Reserve Bank of Richmond Economic Quarterly* **87/2**, 23-44.
- EROSA, A. and M. GERVAIS (2002) Optimal taxation in life cycle economies, *Journal of*

- Economic Theory* **105**, 338-369.
- FUTAGAMI, K, Y. MORITA, and A. SHIBATA (1993) Dynamic analysis of an endogenous growth model with public capital, *Scandinavian Journal of Economics* **95**, 607-625.
- GARCÍA-PEÑALOSA, C. and S.J. TURNOVSKY (2005) Second-best optimal taxation of capital and labor in a developing economy, *Journal of Public Economics* **89**, 1045-1074.
- GAUMONT, D. and D. LEONARD (2003) Endogenous labor, human capital formation and growth in an overlapping generation model, unpublished.
- GERVAIS, M. (2004) On the optimality of age-dependent taxes and the progressive U.S. tax system, unpublished.
- JONES, L.E., R.E. MANUELLI, and P.E. ROSSI (1997) On the optimal taxation of capital income, *Journal of Economic Theory* **73**, 93-117.
- JUDD, K.L. (1985) Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics* **28**, 59-83.
- LUCAS, R.E. and N.L. STOKEY (1983) Optimal fiscal and monetary policy in an economy without capital, *Journal of Monetary Economics* **12**, 55-94.
- MICHEL, P. and P. PESTIEAU (2004) Fiscal policy in an overlapping generations model with bequests-as-consumption, *Journal of Public Economic Theory* **6**, 397-407.
- NOURRY, C. (2001) Stability of equilibria in the overlapping generations model with endogenous labor supply, *Journal of Economic Dynamics and Control* **25**, 1647-1663.
- ORDOVER, J.A. and E.S. PHELPS (1975) The concept of optimal tax in the overlapping generation model of capital and wealth, *Journal of Public Economics* **12**, p-1-26.
- PARK N.H. (1991) Steady-state solution of optimal tax mix in an overlapping generation model, *Journal of Public Economics* **46**, 227-246.
- PERSSON, M., PERSSON. T. and L.E.O. SVENSSON (2006) Time consistency of fiscal and monetary policy: A solution, *Econometrica* **74**, 193-212.
- PERSSON, T. and G. TABELLINI (2000) *Political Economics: Explaining Economic Policy*. Cambridge MA, MIT Press.
- PRESCOTT, E.C., (2004) Why do Americans work so much more than Europeans? *Federal Reserve*

- Bank of Minneapolis Quarterly Review* **28/1**, 2-13.
- RAMSEY, F.P. (1927) A contribution to the theory of taxation, *Economic Journal* **37**, 47-61.
- RAZIN, A., E. SADKA, and P. SWAGEL (2002) The aging and the size of the welfare state, *Journal of Political Economy* **110**, 900-918.
- SEFTON, T. (2004) A Fair Share of Welfare: Public Spending on Children in England, CASE report 25.
- TABELLINI, G. 1991 The politics of intergenerational redistribution, *Journal of Political Economy* **99**, 335-357.
- TURNOVSKY, S.J. (1996) Optimal tax, debt, and expenditure policies in a growing economy, *Journal of Public Economics* **60**, 21-44.
- TURNOVSKY, S.J. (2000) Fiscal policy, elastic labor supply, and endogenous growth, *Journal of Monetary Economics* **45**, 185-210.