Measuring the NAIRU with Reduced Uncertainty: A Multiple Indicator-Common Cycle Approach

by

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Abstract:

Standard estimates of the NAIRU or natural rate of unemployment are subject to considerable uncertainty. We show in this paper that using multiple indicators to extract an estimated NAIRU cuts in half uncertainty as measured by variance and a 29 percent reduction in the confidence band. The inclusion of an Okun's Law relation is particularly valuable. The essential notion is the existence of a common cyclical force driving the macroeconomic variables. Model comparisons based on the use of Bayes factors favor the idea of a common cyclical component.

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1. Introduction

A prerequisite for the conduct of countercyclical macroeconomic policy is to know where we are in the cycle – loosely, are we above or below the NAIRU? Measuring the natural rate and the corresponding cyclical fluctuations of the US economy with reasonable precision poses a significant challenge. Our aim in this paper is to reduce uncertainty about the NAIRU by employing a multiple indicator approach. The essential notion is that the concept of "a business cycle" is meaningful in that there is a single "gap" which drives cyclical behavior across sectors and across variables. By employing a number of indicators jointly we are able to significantly improve the precision of estimates of the NAIRU, reducing uncertainty by about half.

While our primary interest is in reducing uncertainty about the NAIRU, we confront three related issues along the way. The first issue is the need for care in measuring uncertainty for a target, such as the NAIRU, which is itself unobserved. For our purposes the resolution is to use standard models from the literature as benchmarks. The second issue is the so-called "pile-up" problem, where the Kalman filter puts too little weight on the variance of the permanent component. Where the goal is to find point estimates of the NAIRU as an unobserved component, the pile-up problem is an annoyance that has been dealt with by imposing reasonable values on the variance parameters. This solution is unsatisfactory when the goal is to measure uncertainty, because picking a value for the variance comes too close to picking a value for total uncertainty. Fortunately, our multivariate approach seems to eliminate the pile-up problem. The third issue is how to compare NAIRU estimates computed from models which have considerable variation in complexity and are non-nested. We use the Bayes factor approximation based on the Schwarz criterion as discussed in Kass and Raftery (1995)).

We treat the NAIRU as an unobserved component to be estimated by the Kalman filter. Staiger, Stock and Watson (1997a, hereafter SSW (1997a)) points out three sources of uncertainty in a state-space setup. The first source of uncertainty is the model uncertainty arising from incomplete knowledge about the true model. The second source is the parametric uncertainty due to estimation of the parameters of the model from a sample. The final source is unpredictable stochastic shocks to the NAIRU, also called filtering uncertainty.

We begin with a discussion of model uncertainty and identification in the context of trying to predict an unobserved component. Measuring uncertainty about an unobserved component adds a twist that isn't present in the discussion about uncertainty in forecasting an ex post observable variable. Suppose the econometrician is attempting to predict an ex post observable variable. For an observable variable, it is only a mild exaggeration to say that whatever model gives the tightest forecast confidence intervals is the best model. For an unobservable variable the econometrician still wants a tight confidence interval, but confidence intervals are comparable only across models using the same stochastic specification for the unobserved NAIRU and gap. One needs to separate arguments about improved prediction from arguments that are really about appropriate models. In other words, a tight confidence interval for an incorrectly defined NAIRU isn't very useful.[1] However, for any given specification a multiple indicator approach has

[1] If tight confidence intervals alone were a sufficient criterion, then further research to identify the NAIRU would be unnecessary, at least for the United States. The Humphrey-Hawkins Act declared full employment level of unemployment to be exactly 4 percent.

the potential to improve precision.[2] Our solution is to start with standard models in the literature and show that using multiple indicators can significantly reduce the variance.

There are two genres of identification restrictions used in the literature. One set of restrictions describes the statistical behavior of the natural rate, for example that the gap averages zero. The other set of restrictions uses the Phillips curve and identifies the gap as the variable that drives a wedge between expected and realized inflation. In particular, we follow SSW (1997a) in making this distinction. Laubach (2001) showed that the NAIRU uncertainty can be reduced by using both kinds of identifying restrictions. We use Laubach's models as a launching point.

As a first model, suppose the natural rate is constant. A constant natural rate is a straw man rather than a seriously tenable model. (See, for examples of time-varying NAIRU, Summers (1986), Juhn, Topel and Murphy (1991), Gordon (1997, 1998), Shimer (1998), and Ball and Mankiw (2002), to name only a few.) Nonetheless, two different estimates of a constant NAIRU reinforce SSW (1997a)'s conclusion that it is very difficult to estimate the natural rate precisely. First, suppose the identifying restriction is that the gap, unemployment minus the NAIRU, averages zero. Then the constant NAIRU, $\bar{N}_U$, is estimated as the sample mean of unemployment, perhaps weighted to account for serial correlation or heteroskedasticity. In our data a regression of unemployment on a constant with an AR(2) error process gives an $\bar{N}_U$ of 5.90 with a standard error of 0.41. Second, a constant NAIRU can be estimated as a parameter in the Phillips curve, $\bar{N}_\pi$. Using this method the estimate is similar, 5.99 with a standard error of 0.49.

[2] This approach has, of course, been employed in other settings. See for example, Avery (1979).

In what follows we decompose total uncertainty within a given model into the components due to parametric uncertainty and filtering uncertainty. In general the portion due to parametric uncertainty is fairly large, a result which is not surprising given the large degree of uncertainty seen in the constant NAIRU model – where by definition there is no filtering uncertainty. We show that moving to a multiple indicator model reduces both parametric and filtering uncertainty as cross-equation correlations improve the efficiency of estimates.

In section 2 we lay out our benchmark model, provide the estimates and show the results on reduction of filtering uncertainty when moving from a univariate to a bivariate approach. We augment our benchmark model in section 3 by estimating the NAIRU shock variance. Thereafter we extend our model to multivariate frameworks sequentially in section 4 and section 5. We summarize and conclude in section 6.

2. Reduction of filtering uncertainty using multiple indicators

2.1 Specifying the benchmark model

The straw men point estimates in the introduction are essentially identical. Informally one might think of the conclusions from one estimate reinforcing those from the other, where “reinforcing” means reducing uncertainty. In this section we formally combine the two estimates, following Kuttner (1994) and Laubach (2001). This provides a benchmark for the more extensive models given below. It also provides an opportunity to outline the “Bayes factor” technique we use for model comparison.

In modeling the NAIRU, we follow the standard set-up of Gordon (1997), SSW (1997a), Laubach (2001). The following equations form the basic model of NAIRU:

(1) $$\Delta\pi_{C,t} = \beta_C(L)\Delta\pi_{C,t-1} + \gamma_C(L)\left(U_{t-1} - N_{\pi,t-1}\right) + \delta_C X_t + \varepsilon_{C,t}$$

(2) $$U_t \equiv N_{U,t} + g_{Ut}$$

where L is the lag operator, $\Delta\pi_{C,t}$ is the first difference of inflation (calculated using the CPI-all items), U_t and N_t are the observed civilian unemployment rate and the NAIRU at time t. The term X_t denotes a vector of supply shocks and g_{Ut} stands for the unemployment gap. The supply shocks used throughout this paper are a dummy variable for the Nixon price control era and the supply shocks measured by the difference between CPI inflation and food and energy price inflation[3].

Continuing the straw example, we can specify $N_{U,t} = \bar{N}_U$, $N_{\pi,t} = \bar{N}_\pi$, and that g_{Ut} follows an AR(2) and then estimate equations (1) and (2) jointly under the assumption of independent error terms across equations. This gives two independent estimates for the natural rate and associated confidence intervals. Or we can make a meaningfully joint estimate by imposing $N_{U,t} = N_{\pi,t}$.

2.2 Bayes factors for model comparison

In addition to seeing whether joint estimation reduces uncertainty, we want to ask whether it does any particular violence to the data. Frequentist approaches have two problems in this application. First, what should we do if the right-hand side specifications are non-nested? (Although in the straw example the joint estimate is nested within the independent estimates.) Second, classical testing is asymmetric in that strong evidence is required to reject the null hypothesis.

[3] We follow Gordon (1990) to construct the dummies for Nixon era price control and King and Watson (1994) construct the supply shocks. The procedures are also mentioned in SSW (1997a).

The Bayes factor methodology suggested in the statistics literature solves these problems by taking the approach of model comparison rather than hypothesis testing. We end up comparing two models by a posterior probability that one model is favored over the other. Right-hand side specifications need not be nested. What's more, the approximation presented by Kass and Raftery (1995) allows use of non-informative priors for model estimation and equally weighted priors across independent and joint models.

For a set of data D and two models M_I and M_J the *Bayes factor*

$$B_{JI} = \frac{\Pr\left(D \middle| M_J\right)}{\Pr\left(D \middle| M_I\right)}$$

gives the posterior odds and summarizes "the evidence provided by the data in favor of one scientific theory, represented by a statistical model, as opposed to another." (Kass and Raftery (1995, page 777). Kass and Raftery, modifying an earlier suggestion by Jeffries (1961), suggest the following standards for evaluating evidence.

Table 1: Model Comparison Using Bayes Factor

$2\log B_{JI}$	Evidence against M_I
0 to 2	Not worth more than a bare mention
2 to 6	Positive
6 to 10	Strong
>10	Very strong

The Schwarz criterion, defined as

$$S = \ell\left(D \middle| M_J\right) - \ell\left(D \middle| M_I\right) - \tfrac{1}{2}\left(d_J - d_I\right)\log n \approx \log B_{JI}$$

where $\ell(\ )$ is the maximized log-likelihood, d is the number of model parameters, and n is the sample size, gives an approximation to the Bayes factor without specification of an explicit prior. $2S$ can then be used with Table 1 to judge whether the independent or joint is strongly preferred by the data.[4]

2.3 Data, estimation and uncertainty in the constant NAIRU model

We use quarterly data from the first quarter of 1955 to the third quarter of 2003, taken from Fred-II data base of the Federal Reserve Bank of St. Louis and the DRI database. Two lags of unemployment gap and two lags of CPI inflation difference proved to be sufficient in the models. (The selection of lags is based on significance of the last lag.)

Panel A of Table 2 gives the univariate results and Panel B of Table 2 gives results for the constant NAIRU from the bivariate model that allows for model comparison. Joint estimation with a single NAIRU restriction reduces the parametric variance around the NAIRU by 60 percent.[5] The Bayes factor favors the single NAIRU model. Using the standards set out in Table 1, the evidence against the two NAIRU model is in the category ‘positive’.

2.4 A time-varying NAIRU model

If we specify the NAIRU to be a simple random walk (as in SSW (1997a), Gordon (1997) and Laubach (2001)) and specify the gap as following an autoregressive

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[4] $-2S$ is the Bayesian information criterion or BIC.

[5] The asymptotic standard error for $\overline{N}_{\pi}$ is 0.49, but because the estimate of the natural rate is derived by dividing the intercept by $\gamma_C(1)$ the asymptotic approximation is likely to be poor. Following SSW (1997a), we report the ‘Gaussian’ confidence interval at the 95 percent level to be between 4.8 percent and 7.5 percent and illustrated in Figure 1.

process of order two to allow for periodicity in the cycle measure, we have a potentially realistic model. We add equations (3) and (4) to the natural rate model.

(3) $N_t = N_{t-1} + \varepsilon_{N,t}$

(4) $g_{U\,t} = \phi_{U1} g_{U\,t-1} + \phi_{U\,2} g_{U\,t-2} + \varepsilon_{g_U,t}$

Results appear in Table 3. [6] Panel A shows results for univariate estimates of the NAIRU. [7] Panel B shows the bivariate estimate. We used the standard deviation of the NAIRU shock as 0.2 (also used by Laubach (2001) for both estimates. A time-varying, univariate, NAIRU model shows much higher reported total variance than does the constant NAIRU model. Filtering uncertainty is the dominant source of uncertainty about the NAIRU estimates. The estimates show that the parametric variance is only about six percent of the total variance. In the table we also show the point estimates of NAIRU at the beginning of last three decades along with its total standard deviation, parametric standard deviation and filtering standard deviation. The NAIRU estimates and their 95 percent confidence interval are in Figure 2. Figure 2 also illustrates a main point – that the NAIRU is very imprecisely estimated in the univariate model – by showing the large confidence interval of the NAIRU.

[6] Even though we have not changed the set of observables, estimates including (3) and (4) cannot be compared to the previous models using the Bayes factor. Estimation of the Kalman filter where there are nonstationary state variables involves an initialization which affects calculation of the log-likelihood although it does not affect final estimates. For this reason, using the Bayes factor to compare models with different numbers of nonstationary state variables is an unsolved problem.

[7] State-space models using the Kalman filter generally assume normal errors. In principle this is problematic because it implies that the unemployment rate is unbounded. One approach would be to model the log of unemployment and then back out estimate of the level of the natural rate. As a practical matter we found this to be an issue only in the univariate model. In calculating the filtering uncertainty using Monte Carlo methods, we resampled if the standard deviation of the filtering uncertainty turned out to be greater than 3 – which would put a less than zero NAIRU value within the 95 percent confidence interval based on a 6 percent NAIRU. This meant 9 percent resampling in the univariate model but no resampling in the all the following multivariate models. So, the uncertainty in the univariate model might be downward biased, but there is no such bias in the multivariate models.

Joint estimates appear in Panel B. We observe a dramatic decrease in the average total variance coming from a decline in both average parameter variance and average filtering variance. Parametric uncertainty is reduced by a factor of five from the univariate model. But the drop in filtering uncertainty is even greater and most of the uncertainty in the previous model came from filtering, so reduction in filtering uncertainty dominates by being approximately 95 percent of the decline in total variance. The unemployment gap is fairly persistent; the sum of the autoregressive coefficients is 0.92.

In Figure 3, we show the estimates of NAIRU from the bivariate model along with the 95 percent confidence interval. The graph shows that the gap estimates pick up the shaded NBER recessions efficiently. The estimates of the NAIRU show a rise from the mid 1970s and a decline starting in mid 1980s and keeping low throughout 1990s. These features of the natural rate estimate are consistent with studies like Ball and Mankiw (2002), Gordon (1997, 1998), Juhn, Topel and Murphy (1991), SSW (1997a, 1997b), Laubach (2001), Salemi (1999), Shimer (1998) Katz and Kruger (1999), and Summers (1986).

3. The estimated NAIRU shock variance, its standard error and the NAIRU uncertainty from the bivariate model

We now use the bivariate model described above in equations (1) – (4), generalize the variance covariance matrix of the three shocks, $\varepsilon_{C,t}$, $\varepsilon_{N,t}$ and $\varepsilon_{g_U,t}$, and estimate the matrix. This brings two advantages. Firstly, we estimate rather than fix the variance of the shock to the NAIRU. The bivariate model gives us enough cross-equation information to estimate the shock variance covariance matrix without the pile-up problem.[8] Estimation of the shock variance provides a better estimate of filtering uncertainty. The estimation also allows us to have the standard error of the estimated variance of the NAIRU shock – which enters the calculation of parametric uncertainty. The second motivation is due to the Morley, Nelson and Zivot (2003) (hereafter MNZ) result that the estimates of the trend and cycle can be very sensitive to the correlation structure of the shocks.

We faced a computational issue regarding parametric uncertainty while estimating the above model. Estimation of the parameters did not pose any problems but the Hessian of the parameter estimates turned out to be very unstable with respect to some covariance parameter terms between the shocks (log likelihood function very flat for those parameters). We took the following approach to address this problem: we estimated the model with the generalized variance covariance matrix. We noted the off-diagonal parameters with estimated values being close to zero and imprecisely estimated. Then we restricted those off-diagonal parameters to zero and re-estimated the model. The restricted model was used if it was not significantly different at the 90 percent after comparing the log likelihood values. We follow this approach for the rest of the paper.

This effectively meant two restrictions in our model, i.e. the correlation between inflation shock and gap shock and the correlation between inflation shock and natural rate shock were restricted to zero. The log-likelihood difference was not significant at the 75 percent even for one restriction. The results are in Table 4. The standard deviation of the shock to the NAIRU is 0.24, quite close to Laubach and ours imposed value of 0.20. The NAIRU – unemployment gap shock correlation is -0.77, precisely estimated and supports

[8] Elimination of the pile-up problem reflects, presumably, better identifying information. It's not necessarily tied to using a bivariate rather than univariate approach.

the MNZ result. The average total standard error is now 0.57 – a 20 percent rise over the bivariate model in section 2. The Bayes factor favors this estimated standard deviation of NAIRU shock model strongly against the imposed value model (Table 3, Panel B), primarily due to allowing for natural rate and gap shock correlation.

The average parametric standard error doubles, from 0.14 in section 2 to 0.28 in this model, since we now incorporate uncertainty about σ_N which was previously omitted. The filtering uncertainty increases marginally due to a higher value of the variance of the shock to the NAIRU. The NAIRU estimates along with the 95 percent confidence interval are shown in Figure 4. The estimates confirm our previous observations.

4. The Estimation of NAIRU and its uncertainty from four variable models

4.1 The four variable model using GDP and GDP inflation

We now augment the bivariate model in Section 3 to a multivariate model by using two more variables. We have one more inflationary measure based on GDP (chained) deflator and real GDP. The real GDP (in natural logs), Y_t, equation, following Watson (1986), Kuttner (1994), MNZ (2003), is specified as a sum of a permanent stochastic trend, $T_{Y\,t}$ and the output gap, $g_{y\,t}$:

(5) $$Y_t = T_{Y\,t} + g_{Y\,t}.$$

The permanent stochastic trend follows a random walk with a constant drift and the output gap follows a second order autoregressive process:

(6) $$T_{Y\,t} = \mu_Y + T_{Y\,t-1} + \varepsilon_{T_Y\,t}$$

(7) $$g_{Y\,t} = \phi_{Y1} g_{Y\,t-1} + \phi_{Y2} g_{Y\,t-2} + \varepsilon_{g_Y,t}$$

We specify the new inflation equation based on GDP deflator where $\Delta\pi_{G,t}$ is the first difference of the GDP inflation rate depends on its own lags, lagged output gaps and supply shocks. The supply shock based on King and Watson (1994) is measured as a deviation from GDP inflation instead of CPI inflation.

(8) $$\Delta\pi_{G,t} = \beta_G(L)\Delta\pi_{G,t-1} + \gamma_G(L)g_{Y\,t-1} + \delta_G X_t + \varepsilon_{G,t}$$

Finally, following Clark (1989), we link the output gap and the unemployment gap by a dynamic version of Okun's Law:

(9) $$g_{U\,t} = \sum_{k=0}^{K} \theta_{Y,k} g_{Y\,t-k}$$

So, equations (1) – (9) together form a multivariate model of NAIRU estimation where there is a single gap driving the dynamics of cyclical fluctuations in GDP and unemployment. We will refer to this type model as the 'single gap' model. This idea of a single gap or one common cycle is not new in the business cycle literature. Chauvet (1998) used this in a regime switching context, Issler and Vahid (2001) used it in a VAR context[9] and Stock and Watson (1989) used it in a multiple indicator common component context to extract a measure of coincident index of business cycle. We will compare the performance of the single gap model against alternative of using equations (1) - (8) as the model of estimation. This omits the linkage equation (9) and allows two different measures of gap – unemployment gap and output gap. We will denote this type of model as the 'multiple gap' model. Since both models are defined over the same variables but have different number of parameters, the Bayes factor can be used to discriminate between the models.

[9] Interestingly, Issler and Vahid (2001) show that using the common cycle restriction improves the efficiency of the model.

The estimation results are reported in Table 5. Based on the significance of the last lag, we used one lag (along with the contemporaneous) for equation (9). This is consistent with the Clark (1989) framework. For equation (8) we had to use three lags of GDP inflation differences and two lags of the output gap. Comparing the log likelihoods between the single gap and multiple gap models – the multiple gap model has a higher likelihood. However, it also has two more parameters[10] and the Bayes factor 'positively' favors the single gap model. The point estimate of standard deviation of the natural rate shock is lower than our previous estimate but not significantly lower than 0.2. The average total variance of the NAIRU estimates lower than before. This roughly translates to 20 percent shrinkage of the confidence interval around NAIRU when compared with Table 3.

We show the estimates of NAIRU and gaps from single gap and multiple gap models in Figure 5. In the top left panel we have the NAIRU estimates from the single gap model and on the top right panel we the NAIRU estimates from the multiple gap model. They share very similar dynamics even though the confidence interval around the multiple gap NAIRU is wider. Comparing the output gap estimates in the bottom left panel, there is very little difference in between the output gap estimates from two models. Finally, in the bottom right panel we show the unemployment gap estimates along with the output gap from the multiple gap model. They seem to be strongly negatively related with very similar dynamics.

[10] The low and insignificant off-diagonal parameters of the lower triangular matrix based on Cholesky decomposition of the variance covariance matrix of the shocks were restricted to zero using 90 percent significance level.

4.2 The four variable model using employment and wage inflation

We now re-extend the bivariate model in Section 3 to a multivariate model by using two other variables, employment and wage inflation. The employment level (in logs), L_t, equation is a sum of a permanent, stochastic trend, T_{Lt}, and an employment gap variable g_{L_t}:

(10) $L_t = T_{Lt} + g_{L_t}$.

We assume the permanent, stochastic trend of the employment variable, T_{Lt}, follows a random walk with a constant drift:

(11) $T_{Lt} = \mu_L + T_{Lt-1} + \varepsilon_{T_L t}$

The wage inflation equation also has a similar structure as our equation (1) in Section 2 except that we use the employment gap instead of the unemployment gap:

(12) $\Delta\pi_{W,t} = \beta_W(L)\Delta\pi_{W,t-1} + \gamma_W(L)g_{L_t} + \delta_W X_t + \varepsilon_{W,t}$

In the above equation $\Delta\pi_{W,t}$ is the first difference of the wage inflation rate. The supply shock is measured as a deviation from wage inflation. Our crucial element in this four variable model is the specification of the employment gap. As in the model with GDP, the multiple gap specification would imply a separate autoregressive stochastic process for the employment gap:

(13) $g_{Lt} = \phi_{L1} g_{Lt-1} + \phi_{L2} g_{Lt-2} + \varepsilon_{g_L,t}$.

The alternative of a single gap implies either the unemployment gap is dependent on the employment gap or vice versa. Based on log-likelihood values and SICs, the specification that the employment gap is a linear function of contemporaneous and lagged unemployment gap seems a better description of the data. Therefore, the linkage equation specifying the employment gap in the single gap model is:

$$(14) \qquad g_{L\,t} = \sum_{m=0}^{M} \theta_{L,m} g_{U\,t-m} .$$

Equations (1) – (4), (10) – (12) and (14) now form our new single gap multivariate model. The multiple gap model is denoted by equations (1) – (4) and (10) – (13) and omits the linkage equation. Based on significance of last lag, we used contemporaneous and one lag of unemployment gap in equation (14). Similarly, we used three lags of first differences in wage inflation and two lags of employment gaps in equation (12). As before, we start our estimation with a generalized variance – covariance matrix of the shocks and then restrict the off-diagonal parameters.

The estimates of the model are in Table 6. The multiple gap model has two more parameters than the single gap model but the Bayes factor again favors the single gap model. The parameter estimate of the standard deviation of the shock to the NAIRU is 0.19 and precisely estimated. The estimate of the standard deviation of the employment trend shock is moderate and precise but lower than the estimate for GDP trend shock. The correlation of the natural rate shock and the gap shock is negative. The drift term implies a 1.7 percent annual employment growth.

More importantly, we see a similar drop in the NAIRU uncertainty as with GDP although not as large in magnitude. The average total variance drops to 0.24 and this translates to approximately 15 percent reduction in the confidence interval around NAIRU. The top two panels of Figure 6 compare the NAIRU and uncertainty estimates from the single gap and multiple gap model. They show very similar NAIRU dynamics in the two panels and a slightly sharper confidence band in the top left panel depicting the single gap case. The unemployment gap estimates from the two models in the bottom left panel show very similar estimates. Similarly the bottom right panel comparing the unemployment gap and employment gap estimates from the multiple gap model show the expected inverse but very similar dynamics in both gaps.

4.3 Model comparisons of the four variable models

In this section we re-estimate the two four variable multiple gap models but without allowing for any cross correlation between the components. For example, from section 4.1 we estimate the multiple gap model with separate unemployment gap and GDP gap and we do not allow the gaps to be correlated. We treat the two observable variables in each gap estimation as a block with three unobserved components. We then allow the correlation between components within a block to be identical with previous estimation but restrict all the cross block correlations to be zero. This in effect extends the bivariate model of NAIRU estimation to a four variable model without any external influence on the estimation of NAIRU. Since model comparison using Bayes factor is defined over identical observations, we can now compare the no cross correlation four variable variation of the bivariate model to the four variable single gap model.

The results from estimating the two no cross correlation models are in Table 7. The value of Bayes factor in the top panel, computed as evidence against multiple gap model with no cross correlation in favor of the single gap model in Table 5, is 174 – strong evidence favoring the single gap model. The estimates of the standard deviation to the NAIRU shock and correlation of the NAIRU and unemployment gap shock are almost identical to the bivariate model. The bottom panel of Table 7 also shows a similar story using employment and wage inflation. The Bayes factor again is very large and the evidence strongly favors the single gap model. The NAIRU estimates from these two models are shown in Figure 7 along with the NAIRU from the bivariate model for comparison. The estimates are almost identical. Overall, both the models imply that the single gap model brings in important additional information over the bivariate model.

5. The Estimation of NAIRU and its uncertainty from six variable model

In this section we bring together the elements of Section 4 to incorporate the information in the GDP, GDP inflation, employment and wage inflation simultaneously. This results in a six variable model of NAIRU and we will compare the performance of the single gap and the multiple gap models. The multiple gap model is given by equations (1) – (8) and (10) – (13). For the single gap model, we re-specify the employment gap equation as:

$$(15) \qquad g_{L\,t} = \sum_{q=0}^{Q} \theta_{L,q} g_{Y\,t-q} \,.$$

The above equation essentially substitutes equation (9) in equation (14) and reparameterizes the coefficients. Therefore equations (1) – (3), (5) – (12) and (15) make the six variable single gap model. The lags in the equations are chosen as before.

5.1 Estimates from the six variable models

The estimates are given in Table 8. The SIC estimates reconfirm that the single-gap model strikes a better balance between number of parameters and log-likelihood when compared to the multiple gap model. The point estimate of the standard deviation of the NAIRU shock is 0.17 and precise. The standard deviation of the GDP trend shock is higher than in Table 5 but their confidence intervals overlap quite a bit. The same true for the correlation between trend shock and gap shock, which is higher in this model. The standard deviation of the employment trend shock and the drift parameters are very similar to our previous estimates. The Bayes factor strongly favors the single gap model over the multiple gap model.

The average total variance of NAIRU declined further in this model – implying approximately 29 percent reduction in the confidence band over the bivariate model. Overall variance drops in half, from 0.33 to 0.16. This overall drop comes from roughly equal proportional decreases in each component, the parametric variance dropping from 0.08 to 0.03 and the filtering variance dropping from 0.24 to 0.13. Since filtering variance was considerably larger in terms of absolute level, most of the total decrease is due to the reduction in filtering uncertainty. Note that a considerable part of the improvement in the filtering uncertainty is due to the lower estimate of the NAIRU shock variance.

In Figure 8, the NAIRU estimates from the single gap model in the top left panel are quite similar to previous estimates but the 95 percent confidence interval is narrower. The NAIRU estimates from the multiple gap model are in the top right panel[11]. Comparison of output gaps with the multiple gap model in the bottom left panel still shows very similar output gap dynamics in both models. The bottom right panel compares the dynamics of different gaps from the multiple gap model. The employment gap and the output gap show similar dynamics whereas unemployment gap show inverse dynamics to them.

[11] The uncertainty estimates of this multiple gap NAIRU is based on an approximate Choleski decomposition of the variance covariance matrix of the hyper parameters. We had to use the approximation since two diagonal elements of shock covariance matrix were very close to zero with extremely flat likelihood. The approximation is based on Schnabel and Eskow (1990) modified Choleski factorization that

5.2 Estimates from the six variable models with no cross correlation

We now reexamine our evidence from single gap six variable model by comparing it to different variations of two gap models with no cross correlations. To start out, we compare the six variable single gap model with a model where unemployment gap is measured separately and exactly as in the bivariate model. The remaining four variables have a single gap exactly as in the six variable single gap model. We restrict the correlation between the components of the two blocks to be zero. This allows us to use Bayes factor to compare the bivariate model to the single gap six variable model. The estimated Bayes factor of 218 in the top panel of Table 9 is very strong evidence against the two gap model with no cross correlation when compared with the single gap model of Table 8. As before, the estimates of the standard deviation of NAIRU shocks and the NAIRU-unemployment gap correlation exactly match the bivariate estimates.

We follow up by comparing the two single gap four variable models of Section 4 to single gap six variable model by restricting the gaps and the cross correlations. The middle panel of Table 9 compares with the single gap model of Table 5 where employment and unemployment gaps were driven by a single common gap. The Bayes factor of 179 in Table 9, as before, is strong evidence against the two gap model with no cross correlation. The same is true for the case where unemployment gap and GDP gap have a common gap. That Bayes factor is 122, once again strong evidence to favor the single gap model. The estimates of standard deviation of NAIRU shock and the correlation parameters match the corresponding estimates from four variable models. We

minimizes the elements of diagonal matrix added to the original matrix based on iteratively updated Gerschgorin bounds. This is only case in the paper where this approximation was used.

illustrate the NAIRU estimates from all the above three no cross correlation models in Figure 9 along with the estimates from bivariate or four variable models.

6. Conclusion

We show in this paper that using multiple indicators to extract a common unobserved factor helps to reduce the filtering uncertainty and parametric uncertainty around the extracted point estimates. The crucial assumption is that there is a single measure of economic slackness that links different variables. We use this method to estimate the NAIRU and reduce its uncertainty. Specifically, we find that four variables, the GDP deflator, average wage, real GDP, and civilian employment level are valuable indicators of the gap in the business cycle. The improvement in precision reduces the confidence interval by 29 percent. We chose these additional indicators because they did a good job and are consistent with theory. Use of this method opens the possibility for further research which might suggest yet more such useful indicators.

Table 2: Constant NAIRU Estimates and Model Comparison

Panel A: Univariate Estimates

$\bar{N}_\pi$	Parametric Variance	Log L
5.99	0.24	-334.252
$\bar{N}_U$	Parametric Variance	Log L
5.90	0.16	-26.560

Panel B: Bivariate Estimates

$\bar{N}_\pi$	Parametric Variance	$\bar{N}_U$	Parametric Variance	Log L
5.99	0.24	5.90	0.16	-360.812
$\bar{N}$	Parametric Variance	Log L	$2S$ (Evidence against two NAIRUs)	
5.94	0.10	-360.822	5.25	

Table 3: Parameter and NAIRU Estimates from the Univariate and Bivariate Models

Panel A: Univariate Time-varying NAIRU

Log L	$\gamma_C(1)$
-340.11	-0.14

Average Total Variance	Average Parametric Variance	Average Filtering Variance
1.72	0.10	1.62

Date	1980:1	1990:1	2000:1
NAIRU	6.79	5.96	5.49
Total Std. Dev.	1.20	1.20	1.32
Parametric Std. Dev.	0.23	0.17	0.23
Filtering Std. Dev.	1.17	1.18	1.29

Panel B: Bivariate Time-varying NAIRU

Log L	$\gamma_C(1)$	$\phi_U(1)$
-375.20	-0.35	0.92

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.22	0.02	0.20

Date	1980:1	1990:1	2000:1
NAIRU	7.22	6.19	4.99
Total Std. Dev.	0.45	0.44	0.47
Parametric Std. Dev.	0.10	0.07	0.13
Filtering Std. Dev.	0.44	0.44	0.45

Table 4: Parameter and NAIRU Estimates from the Bivariate Model with Estimated NAIRU Shock Variance

Time-varying NAIRU				
Log L	$\gamma_C(1)$	$\phi_U(1)$	σ_N	ρ_{Ng_U}
-365.55	-0.22	0.92	0.24 (0.07)	-0.78 (0.12)

$2S$ (Evidence against imposed standard deviation of 0.2 and no natural rate – gap correlation): 14.03

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.33	0.09	0.24

Date	1980:1	1990:1	2000:1
NAIRU	7.71	6.41	5.42
Total Std. Dev.	0.63	0.51	0.59
Parametric Std. Dev.	0.43	0.21	0.20
Filtering Std. Dev.	0.46	0.47	0.55

Note: The standard errors of the parameter estimates are in the parentheses. There are some rounding-off errors.

Table 5: Parameter and NAIRU Estimates from the Four Variable Model with GDP

Time-varying NAIRU using the Single Gap Model					
Log L	$\phi_Y(1)$	μ_Y	σ_N	σ_{T_Y}	$\rho_{T_Y g_Y}$
-799.75	0.92	0.82 (0.04)	0.17 (0.02)	0.67 (0.05)	-0.15 (0.41)

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.22	0.07	0.15

Date	1980:1	1990:1	2000:1
NAIRU	7.00	6.20	5.06
Total Std. Dev.	0.42	0.414	0.47
Parametric Std. Dev.	0.19	0.22	0.26
Filtering Std. Dev.	0.38	0.38	0.39

Likelihood and BF from the Multiple Gap Model

Log L: -796.93 $2S$ (Evidence against Multiple Gaps): 4.91

Note: The standard errors of the parameter estimates are in the parentheses. There are some rounding-off errors.

Table 6: Parameter and NAIRU Estimates from the Four Variable Model with Employment

Time-varying NAIRU using the Single Gap Model

Log L	SIC	$\phi_U(1)$	μ_L	σ_N	σ_{T_L}	ρ_{Ng_U}
-842.96	9.322	0.92	0.43 (0.02)	0.19 (0.04)	0.30 (0.03)	-0.69 (0.15)

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.23	0.05	0.18

Date	1980:1	1990:1	2000:1
NAIRU	7.86	6.58	5.13
Total Std. Dev.	0.48	0.45	0.51
Parametric Std. Dev.	0.25	0.19	0.21
Filtering Std. Dev.	0.41	0.41	0.47

Likelihood and BF from the Multiple Gap Model

Log L: -840.86 $2S$ (Evidence against Multiple Gaps): 6.35

Note: The standard errors of the parameter estimates are in the parentheses. There are some rounding-off errors.

Table 7: Model Comparisons from the Four Variable Models with No Cross Correlations

Likelihood and BF from the Multiple Gap Model with GDP

Log L	σ_N	ρ_{Ng_U}
-886.84	0.24 (0.07)	-0.78 (0.12)

$2S$ (Evidence against Multiple Gaps with No Cross Correlation): 174.18

Likelihood and BF from the Multiple Gap Model with Employment

Log L	σ_N	ρ_{Ng_U}
-901.71	0.24 (0.07)	-0.78 (0.12)

$2S$ (Evidence against Multiple Gaps with No Cross Correlation): 122.77

Note: The standard errors of the parameter estimates are in the parentheses.

Table 8: Parameter and NAIRU Estimates from the Six Variable Model with GDP and Employment

Time-varying NAIRU using the Single Gap Model

Log L	σ_N	σ_{T_Y}	σ_{T_L}
-1265.85	0.17 (0.02)	0.78 (0.08)	0.28 (0.02)
$\phi_Y(1)$	μ_Y	μ_L	$\rho_{T_Y g_Y}$
0.90	0.82 (0.05)	0.43 (0.02)	-0.51 (0.13)

Average Total Variance	Average Parametric Variance	Average Filtering Variance
0.16	0.03	0.13

Date	1980:1	1990:1	2000:1
NAIRU	7.59	6.73	5.23
Total Std. Dev.	0.39	0.37	0.40
Parametric Std. Dev.	0.16	0.12	0.10
Filtering Std. Dev.	0.35	0.35	0.39

Likelihood and BF from the Multiple Gap Model

Log L: -1263.65 $2S$ (Evidence against Multiple Gaps): 6.13

Note: The standard errors of the parameter estimates are in the parentheses. There are some rounding-off errors.

Table 9: Model Comparisons from the Six Variable Models with No Cross Correlations

Likelihood and BF from the Two Gap Model with Employment and GDP as Single Gap

Log L	SIC	σ_N	ρ_{Ng_U}
-1377.61	15.265	0.24 (0.07)	-0.78 (0.12)

$2S$ (Evidence against Two Gaps with No Cross Correlation): 218.23

Likelihood and BF from the Two Gap Model with Employment and Unemployment as Single Gap

Log L	SIC	σ_N	ρ_{Ng_U}
-1360.74	15.065	0.19 (0.04)	-0.68 (0.16)

$2S$ (Evidence against Two Gaps with No Cross Correlation): 179.21

Likelihood and BF from the Two Gap Model with GDP and Unemployment as Single Gap

Log L	SIC	σ_N	$\rho_{T_Y g_Y}$
-1332.45	14.775	0.17 (0.02)	-0.15 (0.22)

$2S$ (Evidence against Two Gaps with No Cross Correlation): 122.63

Note: The standard errors of the parameter estimates are in the parentheses.

Figure 1: The Constant NAIRU and Its 95 Percent "Gaussian" Confidence Interval

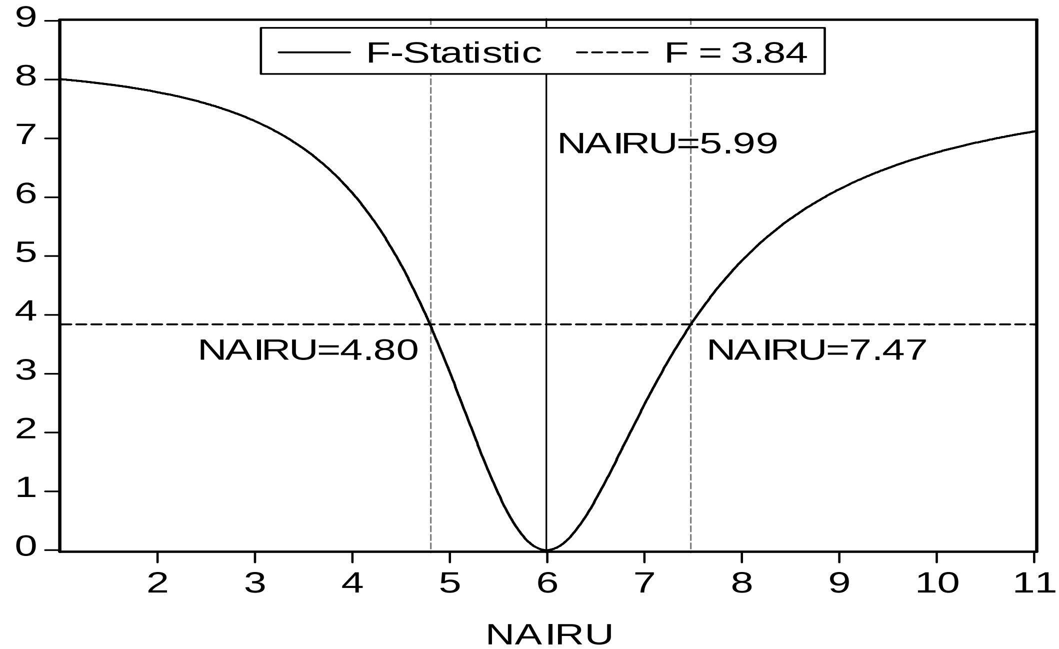


Figure 2: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Univariate Model

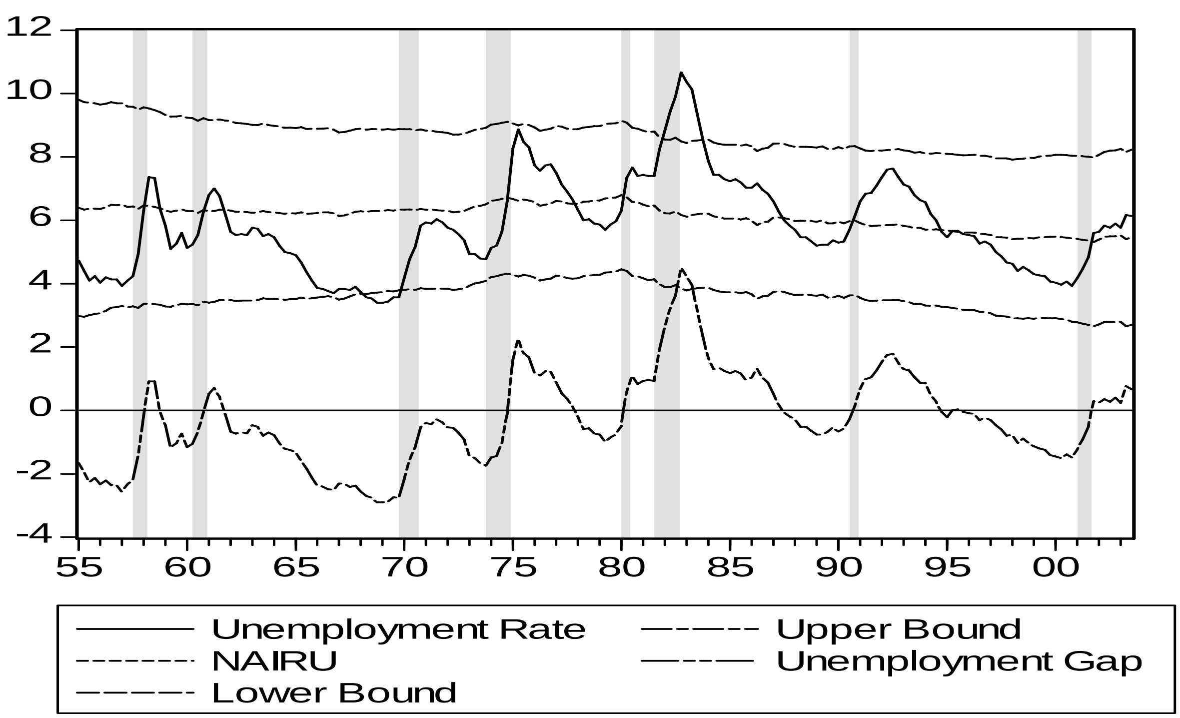


Figure 3: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model

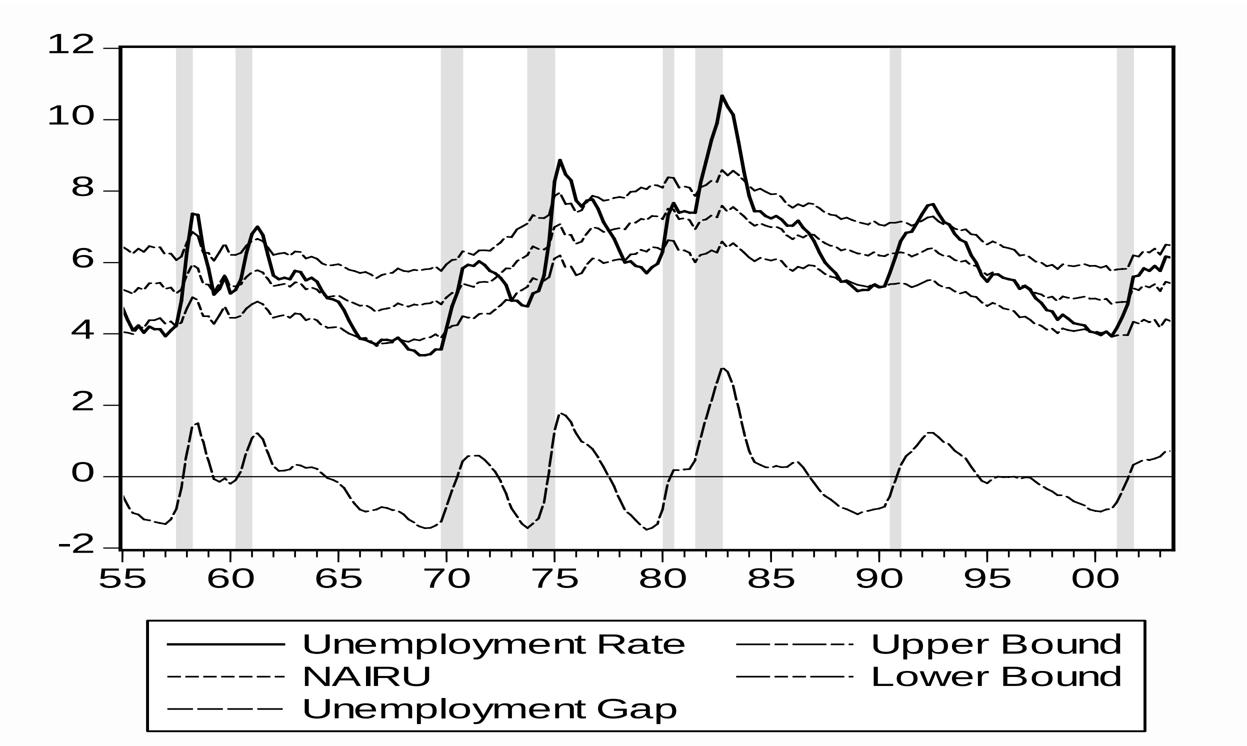


Figure 4: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model with Estimated NAIRU Shock Variance

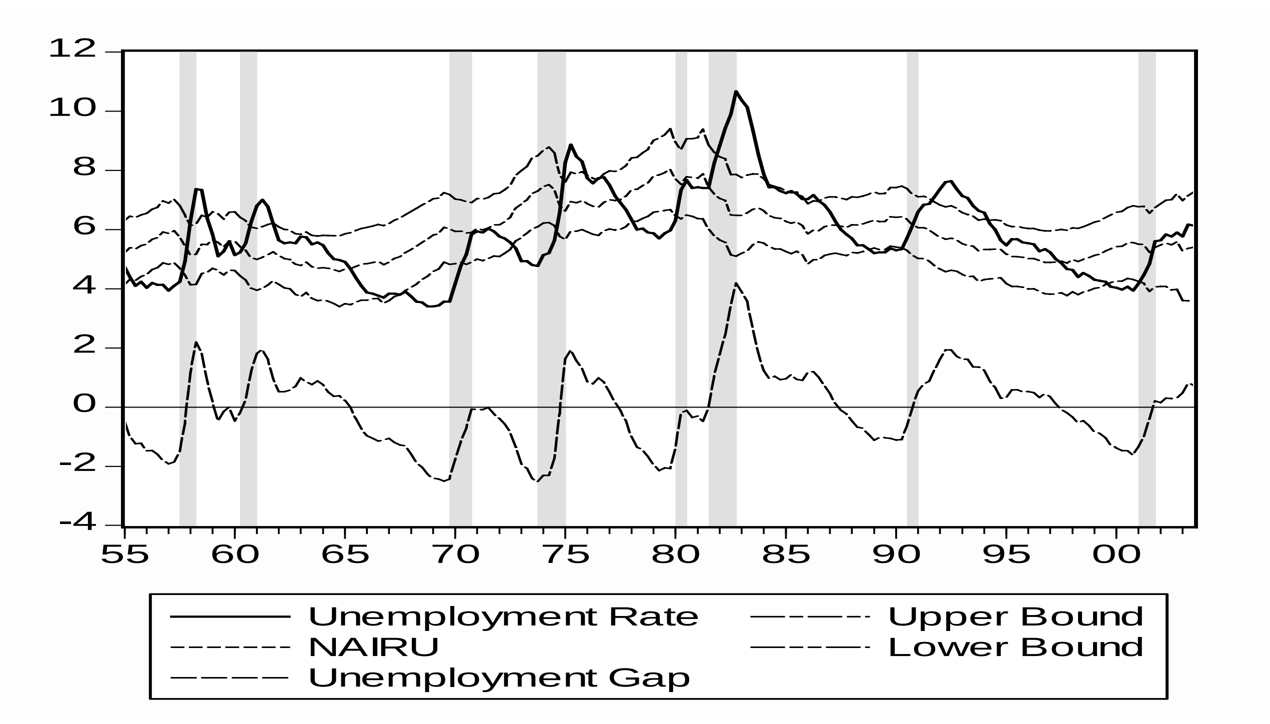
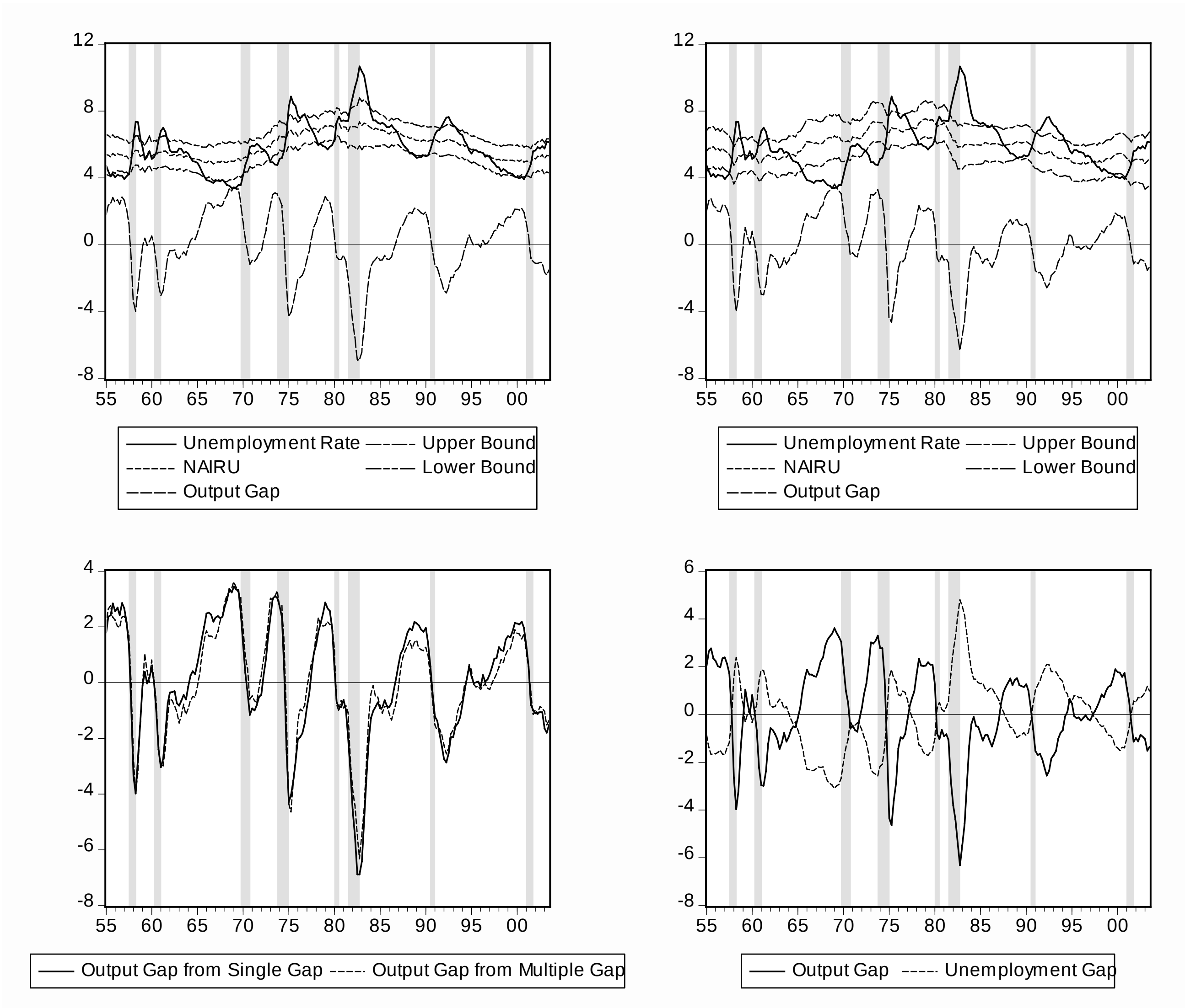
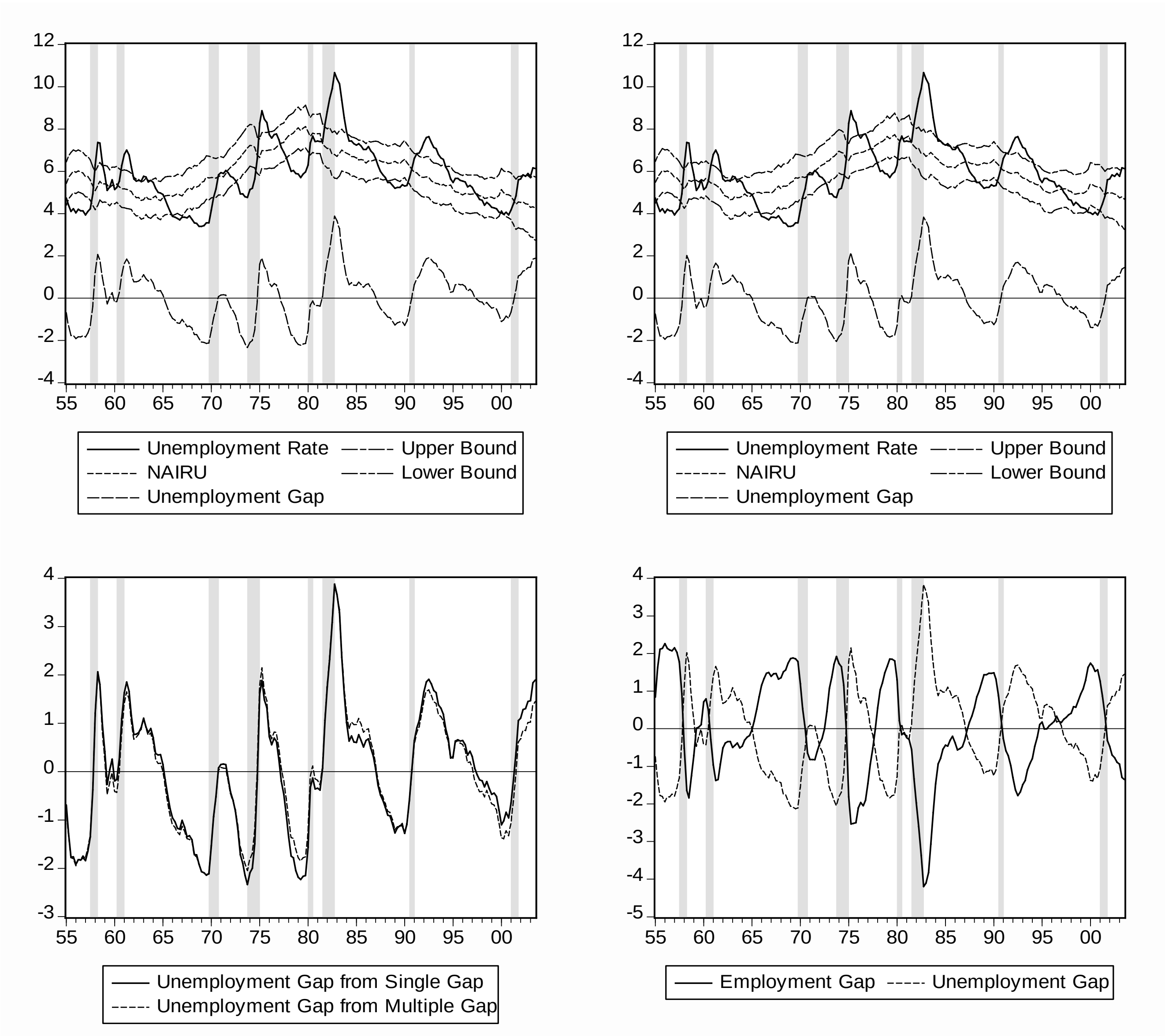


Figure 5: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Four Variable Model with GDP



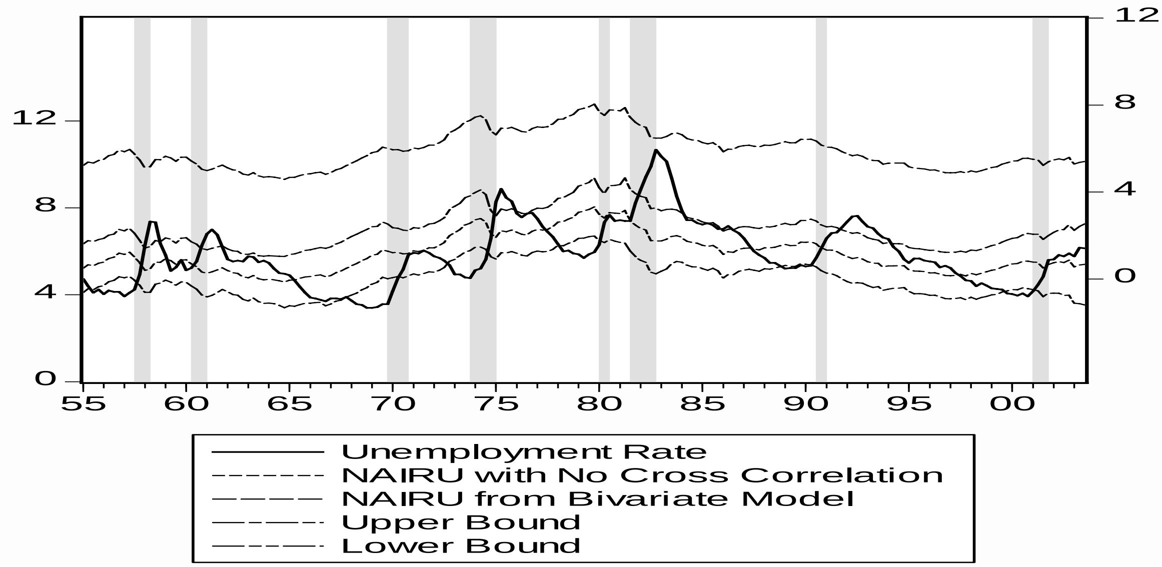
Note: The upper-left panel shows NAIRU estimates and its 95 percent confidence interval from the single-gap model with GDP and GDP inflation. The upper-right panel shows NAIRU estimates and its 95 percent confidence interval from the multiple-gap model. The bottom-left panel compares the output gap estimates from single-gap and multiple-gap models. Finally, the bottom-right panel shows the unemployment gap and output gap estimates from multiple-gap model.

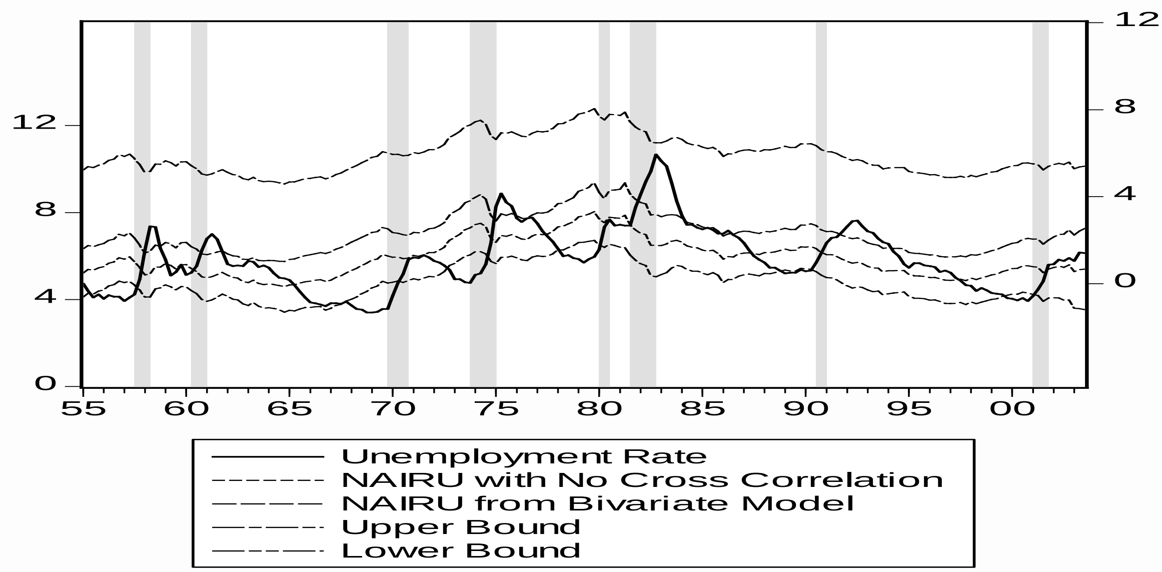
Figure 6: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Four Variable Model with Employment



Note: The upper-left panel shows NAIRU estimates and its 95 percent confidence interval from the single-gap model with employment and wage inflation. The upper-right panel shows NAIRU estimates and its 95 percent confidence interval from the multiple-gap model. The bottom-left panel compares the unemployment gap estimates from single-gap and multiple-gap models. Finally, the bottom-right panel shows the unemployment gap and employment gap estimates from multiple-gap model.

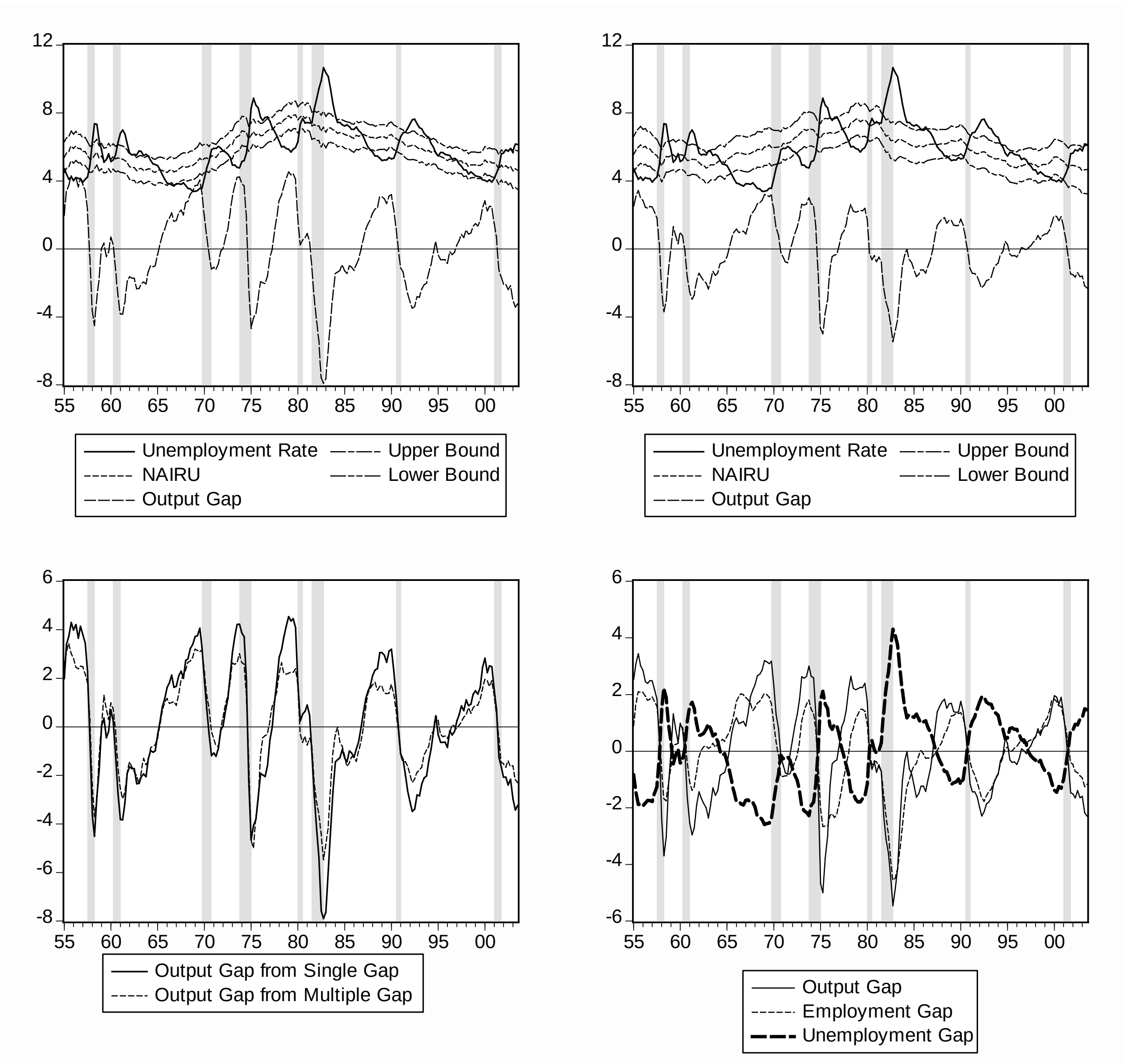
Figure 7: The NAIRU Estimates from Four Variable Models with No Cross Correlations





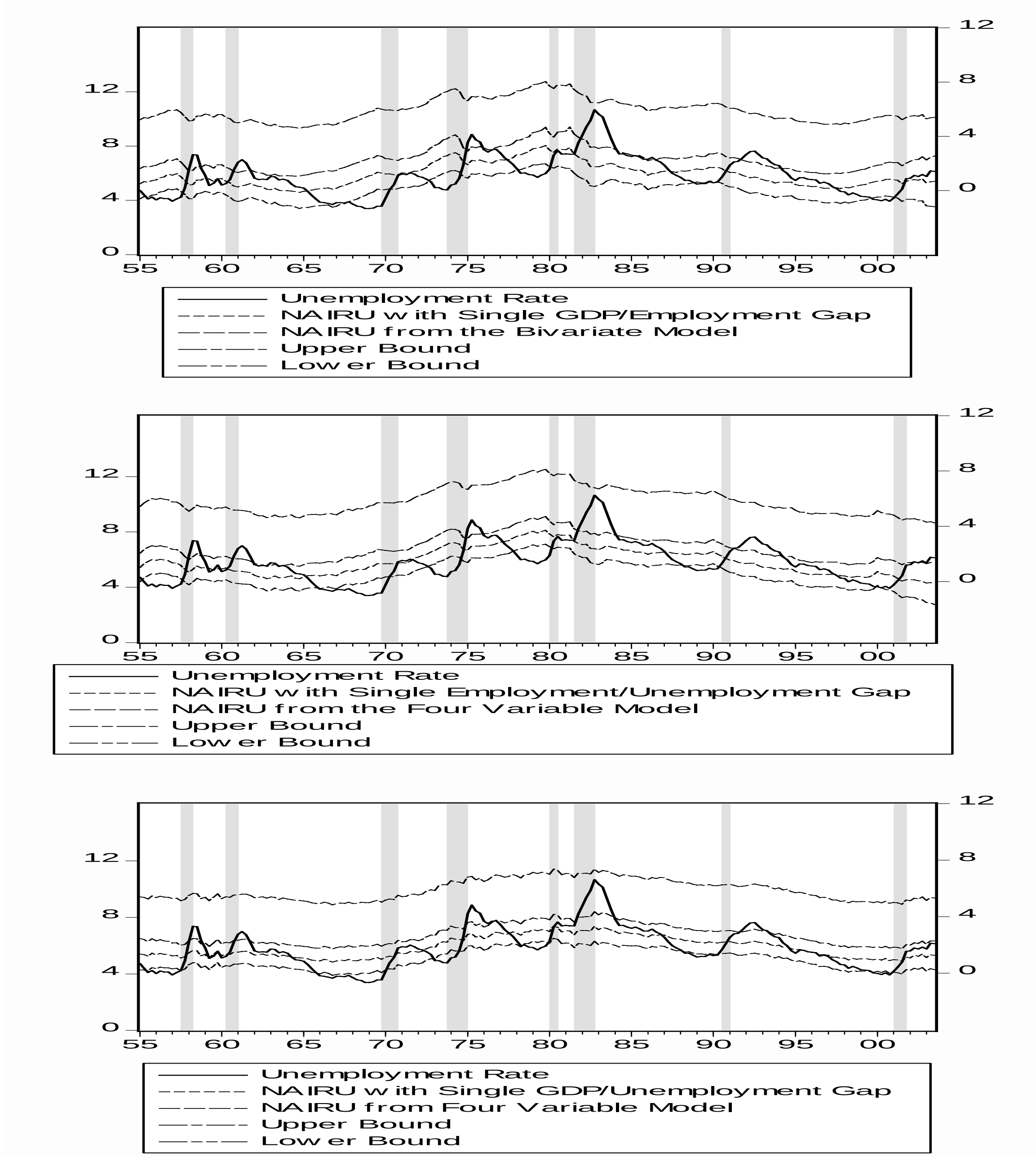
Note: The top panel shows the NAIRU estimates and its 95 percent confidence interval from the four variable model with GDP and GDP inflation. The bottom panel shows the NAIRU estimates and its 95 percent confidence interval from the four variable model with GDP and GDP inflation. In both the panels the NAIRU estimates from the bivariate model is provided on the left scale. They are not allowed overlap with the NAIRU estimates with no cross correlations because the estimates become graphically indistinguishable.

Figure 8: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Multivariate Model with GDP and Employment



Note: The upper-left panel shows NAIRU estimates and its 95 percent confidence interval from the single-gap model with GDP, employment GDP inflation and wage inflation. The upper-right panel shows NAIRU estimates and its 95 percent confidence interval from the multiple-gap model. The bottom-left panel compares the output gap estimates from single-gap and multiple-gap models. Finally, the bottom-right panel shows the output gap, the unemployment gap (thicker and darker line) and the employment gap estimates from the multiple-gap model.

Figure 9: The NAIRU Estimates from Six Variable Models with No Cross Correlations



Note: The top panel shows the NAIRU estimates and its 95 percent confidence interval from the six variable model where unemployment gap is separate from the GDP/employment gap. The middle panel shows the NAIRU estimates and its 95 percent confidence interval from the six variable model where GDP gap is separate from the employment/unemployment gap. The bottom panel shows the NAIRU estimates and its 95 percent confidence interval from the six variable model where employment gap is separate from the GDP/unemployment gap. In all the panels the NAIRU estimates from their corresponding single gap two/four variable models are provided on the left scale. They are not allowed overlap with the NAIRU estimates with no cross correlations because the estimates become graphically indistinguishable.

Bibliography

Apel, Mikael and P. Jansson (1999), “A Theory-consistent System Approach for Estimating Potential Output and the NAIRU”, *Economics Letters*, 64 (3), 271-275.

Avery, Robert B (1979), “Modeling Monetary Policy as an Unobserved Variable,” *Journal of Econometrics*, 10, 291-311.

Ball, Laurence (1999), “Aggregate Demand and Long-Run Unemployment”, *Brookings Papers on Economic Activity* 2, 189-251.

Ball, Laurence and N. Gregory Mankiw (2002), “The NAIRU in Theory and Practice”, *The Journal of Economic Perspectives*, 16, pp. 115-136.

Chauvet, Marcelle (1998), “An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching”, *International Economic Review*, 39(4), 969-96.

Clark, Peter K. (1987); “The Cyclical Component of US Economic Activity”, *Quarterly Journal of Economics*, 102, 797-814.

Clark, Peter K. (1989); “Trend Reversion in Real Output and Unemployment”, *Journal of Econometrics*, 40, 15-32.

Estrella, Arturo, and Frederic S. Mishkin (1998), “Rethinking the Role of NAIRU in Monetary Policy: Implications of Model Formulation and Uncertainty,” *NBER Working Paper* No. 6518.

Giannoni, M.P. (2002), “Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model”, *Macroeconomic Dynamics*, 6(1), 111 – 114.

Gordon, R.J. (1990), “What is New-Keynesian Economics?” *Journal of Economic Literature*, 28(3), 1115 - 1171.

Gordon, Robert J (1997), “The Time-Varying NAIRU and its Implications for Economic Policy”, *Journal of Economic Perspective*s, 11, 11-32.

Gordon, Robert J, (1998), “Foundations of the Goldilocks Economy: Supply Shocks and the Time-Varying NAIRU,” *Brookings Papers on Economic Activity*, 2, 297-346.

Hamilton, James (1986), “A Standard Error for the Estimated State Vector of a State Space Model,” *Journal of Econometrics*, 33, 387-397.

Hamilton, James (1994), “Time Series Analysis,” *Princeton University Press*.

Issler, J. V. and F. Vahid (2001), “Common Cycles and the Importance of the Tarnsitory Shocks to Macroeconomics Aggregates,” *Journal of Monetary Economics*, 47, 449-75.

Jeffries, H. (1961), *Theory of Probability* (3rd ed.), Oxford University Press.

Juhn, Chinhui, Murphy, K., and R. Topel (1991), “Why Has the Natural Rate of Unemployment Increased over Time?” *Brookings Papers on Economic Activit*y, 2, 75 – 126.

Kass, Robert E. and A. Raftery (1995), “Bayes Factors,” *Journal of the American Statistical Association*, 90, 773-795.

Katz, L. and A. Krueger (1999), “The High Pressure U.S. Labor Market of the 1990s,” *Brookings Papers on Economic Activity*, 1, 1-87.

Kim, C-J and C. Nelson (1999), “State-Space Models With Regime Switching: Classical and Gibbs-Sampling Approaches with Applications”, *MIT Press*.

King, R.G., and Mark W. Watson (1994), “The Postwar U.S. Phillips Curve: A Revisionist Econometric History,” *Carnegie-Rochester Conference on Public Policy*, 41, 157 - 219.

Kuttner, Kenneth N. (1994), “Estimating Potential Output as a Latent Variable”, *Journal of Business and Economic Statistics*, 12, 361-368.

Laubach, Thomas (2001), “Measuring the NAIRU: Evidence from Seven Economies,” *The Review of Economics and Statistics,* 83, 218-231.

Laubach, Thomas and John Williams (2003), “Measuring the Natural Rate of Interest,” *The Review of Economics and Statistics*, 85, 1063-70.

Lucas, Robert (1973), “Some International Evidence on Output-Inflation Tradeoffs,” *American Economic Review*, 63 (3), 326-334.

Meyer, L., Swanson and V. Wieland, (2001), “Nairu Uncertainty and Non-Linear Policy Rules”, *American Economic Review Papers and Proceedings*, 226-231.

Morley, J., Nelson, C. and E. Zivot (2003), “Why Are Beveridge-Nelson and Unobserved Component Decompositions of GDP So Different?”, *The Review of Economics and Statistics*, 85(2), 235-243.

Onatski, A., and J. H. Stock (2002), “Robust Monetary Policy Under Model Uncertainty in a Small Model of the US Economy”, *Macroeconomic Dynamics*, 6(1), 85 - 110.

Orphanides, A., Porter, R.D., Reifschneider, D., Tetlow, R., and F. Finan (1999), “Errors in the Measurement of Output Gap and the Design of Monetary Policy”, *Finance and Economics Discussion Series Working Paper 1999-45*, Federal Reserve System Board of Governors, Washington, DC.

Orphanides, A. and John Williams (2003), “Robust Monetary Policy Rules with Unknown Natural Rates”, *Finance and Economics Discussion Series Working paper 2003-11*, Federal Reserve System Board of Governors, Washington, DC.

Rudebusch, G., (2001), “Is the Fed too timid? Monetary policy in an uncertain World”, *Review of Economics and Statistics*, 83, 203-217.

Salemi, Michael K (1999), “Estimating the Natural Rate of Unemployment and Testing the Natural Rate Hypothesis,” *Journal of Applied Econometric*s, 14, 1-25.

Schnabel, Robert B. and Elizabeth Eskow (1990), “A New Modified Cholesky Factorization,” *SIAM Journal of Scientific Statistical Computing*, 11, 6: 1136-58.

Shimer, R. (1998), “Why is the U.S. Unemployment Rate So Much Lower?” in B Bernanke and J. Rotemberg, eds., *NBER Macroeconomics Annual*, MIT Press.

Staiger, Douglas, James H. Stock, and Mark W. Watson (2001), “Prices, Wages and the U.S. NAIRU in the 1990s,” in *The Roaring Nineties: Can Full Employment Be sustained?*, ed., Alan B. Krueger and Robert Solow, New York: The Russell Sage Foundation and The Century Foundation Press.

Staiger, Douglas, James H. Stock, and Mark W. Watson (1997a), “How Precise are Estimates of the Natural Rate of Unemployment?” in *Reducing Inflation: Motivation and Strategy*, eds. C.D. Romer and D.H. Romer, Chicago: University of Chicago Press, 195-246.

Staiger, Douglas, James H. Stock, and Mark W. Watson (1997b), “The NAIRU, Unemployment and Monetary Policy,” *The Journal of Economic Perspective*s, 11, 33-50.

Stock, James H. and Mark W. Watson (1989), "New Indexes of Coincident and Leading Economic Indicators," *NBER Macroeconomics Annual*, eds. Olivier J. Blanchard and Stanley Fischer, Cambridge, Mass.: MIT Press, 351–95.

Stock, James H. (1994), "Unit Roots, Structural Breaks and Trends," *Handbook of Econometrics*, vol. 4, 2739 – 2841.

Stock, James H. and Mark W. Watson (1998), "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of American Statistical Association*, 93, 349-358.

Summers, L. (1986), "Why is the Unemployment Rate So Very High Near Full Employment?", *Brookings Papers on Economic Activity*, (2), 339 -383.

Watson, M. W. (1986), "Univariate Detrending Methods with Stochastic Trends," *Journal of Monetary Economic*s, 18, 49-75.

Not for Publication Appendix: The NAIRU and its Uncertainty Estimates from the Six Variable Multivariate Single Gap Model

Year	Nairu	TU	PU	FU	Year	Nairu	TU	PU	FU
1955:1	5.40	0.46	0.16	0.43	1965:4	4.55	0.39	0.16	0.36
1955:2	5.64	0.46	0.17	0.42	1966:1	4.50	0.39	0.16	0.35
1955:3	5.72	0.45	0.17	0.42	1966:2	4.61	0.39	0.17	0.35
1955:4	6.06	0.45	0.18	0.41	1966:3	4.66	0.39	0.17	0.35
1956:1	5.90	0.44	0.17	0.40	1966:4	4.54	0.40	0.18	0.35
1956:2	6.06	0.44	0.18	0.40	1967:1	4.58	0.40	0.19	0.35
1956:3	5.90	0.42	0.16	0.39	1967:2	4.69	0.41	0.20	0.35
1956:4	5.90	0.43	0.18	0.39	1967:3	4.80	0.40	0.19	0.35
1957:1	5.72	0.42	0.18	0.38	1967:4	4.87	0.41	0.20	0.35
1957:2	5.73	0.42	0.18	0.38	1968:1	4.82	0.42	0.23	0.35
1957:3	5.50	0.41	0.15	0.38	1968:2	4.82	0.42	0.23	0.35
1957:4	5.28	0.39	0.12	0.37	1968:3	4.90	0.44	0.25	0.35
1958:1	5.30	0.40	0.14	0.37	1968:4	4.90	0.44	0.27	0.35
1958:2	5.50	0.40	0.15	0.37	1969:1	5.02	0.45	0.28	0.35
1958:3	5.70	0.38	0.11	0.37	1969:2	5.12	0.45	0.28	0.35
1958:4	5.33	0.38	0.10	0.37	1969:3	5.34	0.46	0.30	0.35
1959:1	5.47	0.38	0.11	0.36	1969:4	5.21	0.45	0.27	0.35
1959:2	5.19	0.38	0.09	0.36	1970:1	5.32	0.42	0.23	0.35
1959:3	5.23	0.37	0.09	0.36	1970:2	5.35	0.41	0.20	0.35
1959:4	5.46	0.38	0.11	0.36	1970:3	5.25	0.39	0.17	0.35
1960:1	5.27	0.38	0.13	0.36	1970:4	5.49	0.39	0.15	0.35
1960:2	5.44	0.38	0.12	0.36	1971:1	5.43	0.40	0.17	0.35
1960:3	5.30	0.38	0.14	0.36	1971:2	5.38	0.39	0.17	0.35
1960:4	5.36	0.40	0.17	0.36	1971:3	5.57	0.39	0.17	0.35
1961:1	5.30	0.40	0.18	0.36	1971:4	5.72	0.40	0.18	0.35
1961:2	5.27	0.40	0.18	0.36	1972:1	5.75	0.39	0.16	0.35
1961:3	5.21	0.39	0.16	0.36	1972:2	5.87	0.39	0.17	0.35
1961:4	5.08	0.38	0.14	0.36	1972:3	5.99	0.40	0.18	0.35
1962:1	4.84	0.38	0.14	0.36	1972:4	6.14	0.41	0.21	0.35
1962:2	4.77	0.38	0.14	0.36	1973:1	6.13	0.42	0.23	0.35
1962:3	4.79	0.39	0.15	0.36	1973:2	6.48	0.43	0.25	0.35
1962:4	4.60	0.39	0.16	0.36	1973:3	6.60	0.44	0.27	0.35
1963:1	4.71	0.39	0.16	0.36	1973:4	6.68	0.44	0.26	0.35
1963:2	4.75	0.39	0.15	0.36	1974:1	6.96	0.43	0.25	0.35
1963:3	4.60	0.39	0.15	0.36	1974:2	6.91	0.45	0.27	0.35
1963:4	4.66	0.39	0.16	0.36	1974:3	6.87	0.42	0.23	0.35
1964:1	4.72	0.38	0.14	0.36	1974:4	6.47	0.38	0.12	0.35
1964:2	4.69	0.38	0.13	0.36	1975:1	6.68	0.37	0.11	0.35
1964:3	4.54	0.39	0.15	0.36	1975:2	6.91	0.37	0.11	0.35
1964:4	4.51	0.39	0.16	0.36	1975:3	6.69	0.37	0.10	0.35
1965:1	4.62	0.38	0.15	0.36	1975:4	6.80	0.37	0.10	0.35
1965:2	4.65	0.38	0.15	0.36	1976:1	6.66	0.37	0.09	0.35
1965:3	4.61	0.38	0.15	0.36	1976:2	6.68	0.37	0.09	0.35

Year	Nairu	TU	PU	FU	Year	Nairu	TU	PU	FU
1976:3	6.82	0.37	0.10	0.35	1987:4	6.55	0.38	0.14	0.35
1976:4	6.95	0.36	0.09	0.35	1988:1	6.57	0.38	0.13	0.35
1977:1	7.04	0.37	0.09	0.35	1988:2	6.48	0.38	0.12	0.35
1977:2	7.02	0.37	0.09	0.35	1988:3	6.54	0.37	0.12	0.35
1977:3	7.16	0.37	0.11	0.35	1988:4	6.58	0.37	0.11	0.35
1977:4	7.33	0.37	0.11	0.35	1989:1	6.58	0.37	0.11	0.35
1978:1	7.41	0.38	0.14	0.35	1989:2	6.59	0.37	0.11	0.35
1978:2	7.37	0.39	0.15	0.35	1989:3	6.50	0.37	0.10	0.35
1978:3	7.59	0.39	0.17	0.35	1989:4	6.68	0.37	0.11	0.35
1978:4	7.68	0.40	0.18	0.35	1990:1	6.73	0.37	0.12	0.35
1979:1	7.84	0.40	0.19	0.35	1990:2	6.57	0.37	0.11	0.35
1979:2	7.71	0.40	0.19	0.35	1990:3	6.50	0.37	0.11	0.35
1979:3	7.86	0.41	0.20	0.35	1990:4	6.41	0.37	0.11	0.36
1979:4	7.89	0.41	0.21	0.35	1991:1	6.34	0.37	0.12	0.36
1980:1	7.59	0.39	0.16	0.35	1991:2	6.27	0.37	0.12	0.36
1980:2	7.77	0.38	0.14	0.35	1991:3	6.10	0.38	0.14	0.36
1980:3	7.85	0.40	0.17	0.35	1991:4	6.04	0.39	0.17	0.36
1980:4	7.67	0.41	0.20	0.35	1992:1	6.05	0.41	0.21	0.36
1981:1	7.79	0.41	0.21	0.35	1992:2	6.10	0.43	0.24	0.36
1981:2	7.71	0.41	0.21	0.35	1992:3	6.08	0.43	0.24	0.36
1981:3	7.24	0.41	0.20	0.35	1992:4	5.98	0.41	0.20	0.36
1981:4	7.32	0.41	0.21	0.35	1993:1	5.86	0.41	0.20	0.36
1982:1	7.21	0.42	0.23	0.35	1993:2	5.90	0.40	0.18	0.36
1982:2	7.24	0.44	0.26	0.35	1993:3	5.80	0.39	0.17	0.36
1982:3	6.93	0.48	0.32	0.35	1993:4	5.70	0.40	0.18	0.36
1982:4	7.17	0.48	0.32	0.35	1994:1	5.80	0.39	0.17	0.36
1983:1	6.86	0.47	0.31	0.35	1994:2	5.68	0.39	0.15	0.36
1983:2	7.11	0.45	0.28	0.35	1994:3	5.75	0.38	0.13	0.36
1983:3	7.12	0.43	0.24	0.35	1994:4	5.70	0.37	0.11	0.36
1983:4	7.01	0.41	0.20	0.35	1995:1	5.48	0.39	0.15	0.36
1984:1	6.96	0.40	0.18	0.35	1995:2	5.45	0.39	0.15	0.36
1984:2	6.80	0.40	0.19	0.35	1995:3	5.38	0.38	0.14	0.36
1984:3	6.84	0.41	0.20	0.35	1995:4	5.28	0.38	0.14	0.36
1984:4	6.81	0.40	0.18	0.35	1996:1	5.20	0.39	0.15	0.36
1985:1	6.73	0.41	0.20	0.35	1996:2	5.29	0.38	0.12	0.36
1985:2	6.68	0.41	0.20	0.35	1996:3	5.15	0.39	0.14	0.36
1985:3	6.66	0.40	0.19	0.35	1996:4	5.22	0.38	0.13	0.36
1985:4	6.57	0.41	0.21	0.35	1997:1	5.25	0.38	0.11	0.36
1986:1	6.50	0.42	0.23	0.35	1997:2	5.17	0.38	0.11	0.36
1986:2	6.68	0.41	0.21	0.35	1997:3	5.18	0.38	0.10	0.36
1986:3	6.63	0.41	0.21	0.35	1997:4	5.01	0.39	0.12	0.37
1986:4	6.70	0.40	0.19	0.35	1998:1	5.04	0.38	0.11	0.37
1987:1	6.74	0.39	0.16	0.35	1998:2	4.85	0.39	0.12	0.37
1987:2	6.67	0.38	0.15	0.35	1998:3	4.94	0.39	0.11	0.37
1987:3	6.60	0.38	0.15	0.35	1998:4	4.95	0.39	0.10	0.37

Year	Nairu	TU	PU	FU
1999:1	4.94	0.39	0.10	0.38
1999:2	4.91	0.39	0.10	0.38
1999:3	4.91	0.39	0.10	0.38
1999:4	5.00	0.40	0.10	0.38
2000:1	5.23	0.40	0.10	0.39
2000:2	5.10	0.40	0.09	0.39
2000:3	5.15	0.41	0.10	0.40
2000:4	5.06	0.42	0.12	0.40
2001:1	5.05	0.43	0.12	0.41
2001:2	4.95	0.44	0.13	0.42
2001:3	4.67	0.44	0.12	0.43
2001:4	4.93	0.45	0.12	0.44
2002:1	4.78	0.47	0.13	0.45
2002:2	4.90	0.48	0.14	0.46
2002:3	4.75	0.50	0.15	0.47
2002:4	4.88	0.51	0.15	0.49
2003:1	4.60	0.54	0.18	0.51
2003:2	4.72	0.56	0.20	0.53
2003:3	4.63	0.58	0.20	0.54

Not for Publication Appendix:

A Simple Example of How to Reduce the Filtering Uncertainty of the State Estimates

Let us consider the following unobserved components model where we observe y_{1t}, y_{2t} and y_{3t}. Our problem is to get estimates and the filtering uncertainty of the unobserved components i_{1t}, i_{2t}, i_{3t} and i_{4t} given that we know the parameters of the model. We assume $i_{jt} \sim iidN(0,1),\ j=1,3,4$ and $i_{2t} \sim iidN(0,\sigma^2)$.

$$\begin{aligned} y_{1t} &= i_{1t} + i_{2t} \\ y_{2t} &= i_{3t} + \gamma_1 i_{2t} \\ y_{3t} &= i_{4t} + \gamma_2 i_{2t} \end{aligned}$$

The common component in the three observed series is the unobserved component i_{2t} and this makes the estimates of i_{2t} to have lower filtering variance. A special case is when $\gamma_1 = \gamma_2 = 0$ and the resulting estimates of i_{2t} will be based on only y_{1t}.

We can write the model in the state-space representation as:

Measurement equations:

$$y_t = HI_t$$

$$y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}, H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & \gamma_1 & 1 & 0 \\ 0 & \gamma_2 & 0 & 1 \end{bmatrix}, I_t = \begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix}$$

Transition equations:

$$\begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} i_{1t-1} \\ i_{2t-1} \\ i_{3t-1} \\ i_{4t-1} \end{bmatrix} + \begin{bmatrix} i_{1t} \\ i_{2t} \\ i_{3t} \\ i_{4t} \end{bmatrix}$$

$$i.e., I_t = FI_{t-1} + I_t$$

$$Let, \quad F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In the above transition equations, F is the transition matrix of the state variables and Q is the variance covariance matrix of the shocks to the state variables (which are variances of the states themselves in this special case since the states are all white noise). To start the Kalman filter iteration, we need the steady-state values of I_t as $I_{0|0}$ and uncertainty around I_t at time zero as $P_{0|0}$. We specify those initial values as

$$I_{0|0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$P_{0|0} = FP_{0|0}F' + Q = Q$$

Starting the Kalman filter iteration, we have

$$I_{t|t-1} = FI_{t-1|t-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q = Q$$

We denote $I_{t|t-1}$ as the linear projection of the state variables based on time $t-1$ information. The uncertainty (or the variance-covariance matrix) around the projections is denoted as $P_{t|t-1}$, also based on time $t-1$. The forecast errors are denoted as $\eta_{t|t-1}$, and $f_{t|t-1}$ is the conditional variance of the forecast errors. In the case we are considering

$$\eta_{t|t-1} = y_t - HI_{t|t-1} = y_t$$

$$f_{t|t-1} = HP_{t|t-1}H' = HQH' = \begin{bmatrix} 1+\sigma^2 & & \\ \gamma_1\sigma^2 & 1+\gamma_1^2\sigma^2 & \\ \gamma_2\sigma^2 & \gamma_1\gamma_2\sigma^2 & 1+\gamma_2^2\sigma^2 \end{bmatrix}$$

Updating the iterations to include time t information, we have the Kalman gain component, K_t:

$$K_t = P_{t|t-1}H'f_{t|t-1}^{-1}$$

Therefore

$$I_{t|t} = I_{t|t-1} + K_t\eta_{t|t-1} = K_t y_t$$

$$P_{t|t} = P_{t|t-1} - K_t HP_{t|t-1} = Q - K_t HQ$$

Since the F matrix is a zero matrix, we have $P_{t|t} = P_{t|T}$, where

$$P_{t|t} = P_{t|T} = \begin{bmatrix} \frac{\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & & & \\ \frac{-\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & & \\ \frac{\gamma_1\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{-\gamma_1\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_1^2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \\ \frac{\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{-\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_1\gamma_2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} & \frac{\gamma_2^2\sigma^2}{1+\gamma_1\sigma^2+\gamma_2\sigma^2+\sigma^2} \end{bmatrix}$$

It is obvious from the above matrix that for $\gamma_1 = \gamma_2 = 0$, the variance of $i_{2t|T} = \frac{\sigma^2}{1+\sigma^2}$ is maximum. So, non-zero values of γ_1 and γ_2 will reduce the variance – filtering uncertainty improves with a multiple indicator - common factor approach. Moreover, the marginal effect of σ^2 is

$$\frac{\partial(\text{var}(i_{2t|T}))}{\partial\sigma^2} = \frac{1}{(1+\gamma_1^2\sigma^2+\gamma_2^2\sigma^2+\sigma^2)^2} \geq 0\,.$$

As evident from the analysis above, the same argument applies to the precision of i_{1t}. This example highlights, ceteris paribus, the role a common factor approach can play by extracting information from multiple indicators in improving its precision.

The above model also shows that impact of an additional indicator on improving filtering uncertainty goes down with increasing number of indicators if everything else is same. The reduction in filtering uncertainty when we augment the univariate model to a bivariate system is $\frac{\gamma_1^2\sigma^4}{(1+\sigma^2)(1+\gamma_1^2\sigma^2+\sigma^2)}$. Similarly, when we extend the bivariate to a trivariate system, the decline in filtering uncertainty is $\frac{\gamma_2^2\sigma^4}{(1+\gamma_1^2\sigma^2+\sigma^2)(1+\gamma_1^2\sigma^2+\gamma_2^2\sigma^2+\sigma^2)}$. To simplify the algebra, let us assume that $\gamma_1=\gamma_2=\gamma$ - thereby making the assumption that y_{2t} and y_{3t} individually contain same amount of information about i_{2t}. Then,

$\frac{\gamma^2\sigma^4}{(1+\sigma^2)(1+\gamma^2\sigma^2+\sigma^2)} \geq \frac{\gamma^2\sigma^4}{(1+\gamma^2\sigma^2+\sigma^2)(1+2\gamma^2\sigma^2+\sigma^2)}$. So, in the above model, the biggest reduction in the filtering uncertainty comes from extending the model from the univariate to the bivariate setup.

Not for Publication Appendix:

The Bivariate Model with Generalized Covariance Matrix of the Shocks

In this section we re-estimated the bivariate model of the Section 3 (eqs. (1) – (4)) with a generalized variance covariance matrix of the three shocks. Specifically,

$$Var - Cov(\varepsilon_{C,t}, \varepsilon_{N,t}, \varepsilon_{g_U,t}) = \begin{bmatrix} \sigma_C^2 & & \\ \sigma_{CN} & \sigma_N^2 & \\ \sigma_{Cg_U} & \sigma_{Ng_U} & \sigma_{g_U}^2 \end{bmatrix}$$

is the new variance – covariance matrix of the shocks. This implies estimation of two more parameters and examining the effects of their standard errors on the total uncertainty.

In Table A1, we show the estimation results of the new bivariate model with generalized covariance matrix. The estimate of the standard deviation of the shock to the NAIRU is 0.22, very similar to what studies like Gordon (1997), Laubach (2001) used before and what we have in our Table 3. The estimate of the correlation between the NAIRU and the unemployment gap, ρ_{Ng_U} , is -0.77 – strongly negative like the MNZ result. The comparison of the log likelihood values with Table 3 indicates that the inclusion of the two new parameters were insignificant at the 90 percent level. The estimate of persistence of the unemployment gap is quite similar to the previous estimate, 0.92.

The surprising element of the generalized covariance matrix is in its effect on the total uncertainty. The average filtering variance remain almost the same as in Table 3, not a surprising result given the estimate of the standard deviation of the shock to the NAIRU is quite similar to what used before. However, the average total variance now has risen to 1.33, primarily due to the big rise in the average parametric variance. In Figure A1, we show the new NAIRU estimates along with the new 95 percent confidence bands. The NAIRU estimates are similar to our previous estimates but the confidence bands are a lot wider. This was happening because the log likelihood function is very flat with respect to the two new covariance parameters, making the Hessian and the variance – covariance matrix of the estimated parameters very unstable and resulting in a large increase in the parametric uncertainty.

Table A1: Parameter and NAIRU Estimates from the Bivariate Models with Generalized Variance Covariance Matrix of the Shocks

Time-varying NAIRU (Bivariate)				
Log L	γ_C (1)	ϕ_U (1)	σ_N	ρ_{Ng_U}
-364.97	-0.22	0.92	0.22 (0.10)	-0.77 (0.12)

Average Total Variance	Average Parameter Variance	Average Filtering Variance
0.39	0.16	0.24

Date	1980:1	1990:1	2000:1
NAIRU	7.36	6.31	5.41
Total Std. Dev.	0.86	0.56	0.63
Parametric Std. Dev.	0.74	0.33	0.23
Filtering Std. Dev.	0.45	0.46	0.57

Figure A1: The Time-Varying NAIRU and Its 95 Percent Confidence Interval from the Bivariate Model with Generalized Covariance Matrix of the Shocks