Non-Exponential Discounting: A Direct Test And Perhaps A New Puzzle

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Abstract

Standard models of intertemporal utility maximization under uncertainty assume that agents discount future utility flows at a constant compounded rate—exponential discounting. Euler equations estimated over different time horizons should have equal discount rates. They do not. Rising term yield premia on safe nominal bonds imply discount rates that rise with longer horizons, as uncertainty is much too small to account for the difference in interest rates. Such deviations from exponential discounting are large enough to make a large difference in consumption choices over long horizons. Our rejection of exponential discounting raises doubts about dynamic consistency in consumer choice, and therefore calls into question an underpinning of many intertemporal models. Our results can be viewed as providing estimates of horizon-specific discounts, or alternatively as a further puzzle concerning the linkage between intertemporal substitution and uncertainty.

Keywords: Intertemporal consumer choice, discounting, hyperbolic discounting, consumption,

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Introduction

In the canonical model of dynamic, intertemporal utility maximization, agents are assumed to discount future flow utility exponentially, so that the contribution of consumption τ periods in the future to today's utility is $e^{-\delta(\tau-t)}EU(C_{t+\tau})$. If the future is not discounted exponentially, much of what we think we know about intertemporal dynamics in macroeconomics and in finance is open to question. In his seminal article, Strotz (1955) shows that this assumption is necessary for dynamic consistency. "An individual who … does not discount all future pleasures at a constant rate…finds himself continuously repudiating his past plans." (p. 173). The force of Strotz' argument is so strong that almost all work involving intertemporal choice assumes exponential discounting, the well-known exception being the literature on what is generically called "hyperbolic discounting."

We estimate the standard model of the consumption/investment tradeoff. What is nonstandard is that we estimate the tradeoff across various time horizons, allowing us to ask directly whether or not future pleasures are discounted at a constant rate.

We ask whether agents exponentially discount utility across significant horizons, in particular comparing one-quarter decisions to five-year decisions. If utility one quarter away is discounted by $e^{-.25\times\delta^{(.25)}}$ and utility five years out is discounted by $e^{-5\times\delta^{(5)}}$, does $\delta^{(.25)} = \delta^{(5)}$? The answer appears to be "no." What is more, at long horizons the estimated difference between $\delta^{(.25)}$ and $\delta^{(5)}$ is large enough to be economically important. Our approach is completely standard, except that we look at tradeoffs over long horizons directly instead of cumulating sequences of one-period decisions out over several years. In principle such tradeoffs depend on both long-horizon risk and expected return. To give away the "punch line" of the paper, empirically risk turns out to be pretty much irrelevant and the higher expected returns available on average for longer holding periods imply higher long-horizon discount rates.

Preferences about intertemporal tradeoffs across various horizons are revealed by agents' decisions to invest in (relatively safe) government bonds of corresponding maturities. Yields are typically higher on longer-maturity bonds. These increasing yields, the yield premia, imply increasing discount rates, the discount premia. In other words, $\delta^{(5)} > \delta^{(.25)}$. This may seem a surprising claim, seeming to turn on its head the customary procedure in which one maintains the assumption of exponential discounting and attributes higher long-term yields to greater risk. Perhaps it should not be surprising, since at least since Backus, Gregory, and Zin (1989), it has been known that the customary procedure has great difficulty in accounting for term premia without abandoning the additively separable, expected present value of utility model. As a matter of form, our statistical estimates allow for both non-exponential discounting and a response to risk. But as a practical matter, the effect of the former overwhelms the latter. When long-term government bonds are held to maturity, the only relevant sources of uncertainty are unanticipated inflation and surprises which affect desired consumption growth. Conditional on these two sources of uncertainty, the risk of short-run fluctuations in long-run bond prices does not directly enter the consumer's optimization.[1] Neither source of uncertainty is very large, and the effect of greater uncertainty over long horizons than over short horizons is

[1] One usually thinks of long bonds as being risky because the one-period holding-period return can be volatile. Nothing in our model is inconsistent with looking at one-period returns. But in the canonical model an Euler equation of any horizon is equally valid.

to increase estimates of the deviation from exponential discounting. Risk preferences do matter in these calculations, although less than might be expected because that part of uncertainty that evolves according to a random walk drops out of the model. We use an insight from Weil (1989), positive discount rates constrain the coefficient of relative risk aversion, to bound risk preferences.

We can summarize our results in two parts. The first finding is that the null of exponential discounting is soundly rejected. The second finding consists of estimates of an alternative, nonexponential, discount schedule. We find that the distant future is discounted more heavily than is the near future.[2] The difference is large enough that projections of short-run tradeoffs to long-run decisions are likely to lead to considerable error. In the next section we work through this intuition quantitatively, presenting a calibration based on long-run averages. The section following provides econometric estimates based on the usual Euler equation estimation. We then turn to several robustness checks.

We estimate "the standard model" of intertemporal substitution for a representative consumer using aggregate data. Usually these models look at one-period growth in marginal utility set against one-period returns, r_t, and the discount rate δ, $E[e^{(r_t-\delta)}U'(C_{t+1})/U'(C_t)] = 1$. Our departure from the norm is simply to look at horizons t versus $t+m$ as well as at t

[2] We find that discount rates between periods at distant horizons are higher than discount rates between periods at near horizons, a result at odds with the experimental literature on non-exponential discounting. (See Angeletos et. al. (2001) for a long list of references.) For example, Laibson (1997) writes "Research on animal and human behavior has led psychologists to conclude that discount functions are approximately hyperbolic [Ainslie 1992]. Hyperbolic discount functions are characterized by a relatively high discount rate over short horizons and a relatively low discount rate over long horizons." (page 445). Angeletos et.al. (2001, p. 50) reiterate Laibson's (1997) statement, writing "the experimental evidence implies that the actual discount function declines at a greater rate in the short run than in the long run." The difference between our discount estimates and quasi-hyperbolic discounting is discussed below.

versus $t + 1$.[3] Our estimates share all the usual merits and demerits of extracting preference parameters from aggregate data. One issue is special: Our estimates of the discount schedule are valid under the null of exponential discounting, but are not necessarily valid under the alternative. The distinction has more bite than in many situations. We estimate the standard Euler equation across different horizons. But if there is non-exponential discounting, it is not at all clear that the standard Euler equations apply. So a logically consistent reading of our results allows for discarding exponential discounting without necessarily accepting our alternative estimates of discount rates.

In the body of the paper, we report alternative estimates of discount rates and take them to be meaningful. Other interpretations are possible, and the evidence is more clear in rejecting the null than it is decisive in choosing among alternatives. One (distressing) interpretation is that the dynamic programming/Euler equation model, which is ubiquitous in short-run models in macroeconomics and finance, is a useful approximation to preferences at short horizons but not at intermediate and longer horizons. Preferences that allow for dynamic programming are, after all, a subset of all possible preferences—albeit a very useful subset. Under the slightly more general Strotz (1956) and Pollak (1968) assumptions that the current period agent "honors" the preferences of later period agents, Barro (1999) has shown that under log utility, non-exponential discount rates ought not show up in the data.[4] Since

[3] Singleton (1990) in his *Handbook* chapter estimates the same equation that we do, although for very short horizons. He isn't looking for non-exponential discounting and the discount rates he reports in his Table 12.6 do not appear to be significantly different from one another. Note, however, that the large changes he finds in the estimates of the intertemporal elasticity of substitution might be picking up a non-exponential discounting effect.

[4] Strotz (1956) and Pollak (1968) assume that the consumer in the current period solves a dynamic programming problem that accounts for the solution she will later solve with changed preferences. Barro (1999) writes "With no commitment ability and log utility, the equilibrium exhibits a constant effective rate of time preference and is observationally equivalent to the standard model." Luttmer and Mariotti (2003) show this result also attains

deviations from exponential discounting do show up in the data, one can ask if this is evidence against the Strotz/Pollak formulation. Or perhaps time preferences are better represented by some version of the "preferred habitat" model of Modigliani and Sutch (1966, 1967).

A different approach to the alternative is that we are seeing a failure to sort out time preference versus risk preference rather than short versus long maturity time preference. This would suggest that the alternative relates to the still sought after resolution to the equity premium puzzle (or perhaps better, the "risk-free rate puzzle") in the sense that we rely on the canonical model and the equity premium puzzle raises doubts about that model (see Mehra and Prescott (1985, 2003) or the survey by Kocherlakota (1996)). However, the measure of risk in the equity premium puzzle is the risk to one-period returns. We also measure the risk to longer-term tradeoffs and show that long-term nominal bonds have relatively little long-term risk (perhaps unsurprisingly).[5] This suggests that an alternative needs to not only separate attitudes toward risk and return, but also attitudes toward short-horizon and long-horizon risk.

With this preamble we present our estimates of nonexponential discount rates, recognizing that some readers may prefer to identify the results as a "nonexponential discount rate puzzle."

"when endowments are such that expected utility growth is constant," but not more generally. All this means that there are circumstances under which consumers have non-exponential discount rates that our method would fail to reveal. Empirically, we do find non-exponential discounting, implying either that the Strotz/Pollak assumption doesn't apply or that utility is far enough from logarithmic to be detectable.

[5] Uncertainty also plays a role in the extent to which asset prices can reveal information about non-exponential preferences. If future returns are certain, then if the strong form of the expectations hypothesis does not hold the consumer will attempt to arbitrage between short and long rates without regard to her rate(s) of discount (Kocherlakota (2001)). Slightly more generally, if either sequential short-term or long-term investments first-order stochastically dominate, then, again, the consumer will choose the superior return without regard to discount rates. Under these circumstances the yield curve provides no information about discount rates. Apparently, neither condition pertains in the data.

Model and Economic Estimates

In the canonical model, at time t the representative agent maximizes discounted utility. Let C_{t+m} be real consumption m periods hence. The applicable discount rate is $\delta^{(m)}$, where we use the parenthetical (m) to distinguish a symbol applicable over an m-year horizon. Under exponential discounting, $\delta^{(m)} = \delta \ \forall m$.

The objective function is

$$\int_{m=0}^{m=\infty} e^{-\delta^{(m)} m} EU(C_{t+m}) dm \tag{1}$$

The consumer can save one nominal dollar today, invest it at the certain m-period nominal interest rate today, $r_t^{(m)}$, and increase nominal spending m periods hence by $e^{r_t^{(m)} m}$. While the Euler solution characterizing optimal behavior is usually written for the one-period consumption tradeoff, it is equally valid for all horizons[6]. Hence,

$$E\left[\frac{U'(C_{t+m})/P_{t+m}}{U'(C_t)/P_t}\right] = e^{\left(\delta^{(m)} - r_t^{(m)}\right) m} \tag{2}$$

Suppose we now assume CRRA felicity, $U(C) = \frac{C^{1-\alpha} - 1}{1-\alpha}$, $U'(C) = C^{-\alpha}$. Let $g^{(m)}$ be consumption growth over m periods ($g_t^{(m)} m \equiv \log(C_{t+m}) - \log(C_t)$) and $\pi_t^{(m)}$ be inflation, then we can write equation (2) in the more specific form

$$E\left[\left(e^{g_t^{(m)} m}\right)^{-\alpha} \times e^{-\pi_t^{(m)} m}\right] = e^{\left(\delta^{(m)} - r_t^{(m)}\right) m} \tag{3}$$

[6] Campbell (1986) and Harvey (1988) both make use of this result. See also Singleton (1990).

If the bracketed term in (3) ,is approximately lognormal, then the left-hand side is $e^{\mu+\frac{\sigma^2}{2}}$, where $\mu^{(m)}$ and $\sigma^2_{(m)}$ are the mean and variance of the process $-\alpha g_t^{(m)} m - \pi_t^{(m)} m$. Taking expectations in equation (3) and then taking logs, the relation between discount rates and interest rates is

$$\delta^{(m)} = -\alpha \mathrm{E}\left(g_t^{(m)}\right) + \left[r_t^{(m)} - \mathrm{E}\left(\pi_t^{(m)}\right)\right] + \frac{\sigma^2_{(m)}}{2m} \tag{4}$$

Equation (4) enters our calculations in two ways, levels and differences. First, when m is one period ($m = 0.25$ in our quarterly data), equation (4) gives the relation between the discount rate δ, the real interest rate $r - \mathrm{E}(\pi)$, consumption growth g, and uncertainty σ^2. Imposing the non-negativity on the level of the left-hand side, $\delta > 0$, places a limit on the admissible values of α are admissible in equation . Second, we use the long-horizon versus short-horizon difference in equation (4) to compute the discount premium $\delta^{(m_L)} - \delta^{(m_S)}$, as in equation (5).

$$\begin{aligned}
&\left(\delta^{(m_L)} - \delta^{(m_S)}\right) \\
&\quad = -\alpha\left(E\left(g_t^{(m_L)}\right) - E\left(g_t^{(m_S)}\right)\right) \\
&\quad + \left(\left[r_t^{(m_L)} - \mathrm{E}\left(\pi_t^{(m_L)}\right)\right] - \left[r_t^{(m_S)} - \mathrm{E}\left(\pi_t^{(m_S)}\right)\right]\right) \\
&\quad + \left(\frac{\sigma^2_{(m_L)}}{2m_L} - \frac{\sigma^2_{(m_S)}}{2m_S}\right)
\end{aligned} \tag{5}$$

Temporarily, pretend there is no uncertainty and replace the expectations in equation (4) with their long-run averages, $\bar{g}$, $\bar{r}^{(m)}$, and $\bar{\pi}$. Setting $\delta^{(.25)} > 0$ and solving gives $\alpha <$

$(\bar{r}^{(.25)} - \bar{\pi})/\bar{g}$. Using the data in Table 1 we find $\alpha < .807$.[7] If we continue to ignore uncertainty and use long-run averages, we calculate the discount premium as

$$\delta^{(m_L)} - \delta^{(m_S)} = -\alpha\left(\overline{g^{(m_L)}} - \overline{g^{(m_S)}}\right) + \left[\left(\overline{r^{(m_L)}} - \overline{r^{(m_S)}}\right) - \left(\overline{\pi^{(m_L)}} - \overline{\pi^{(m_S)}}\right)\right]$$

Because calculation of long run average growth and inflation is independent of the horizon, e.g. $\overline{g^{(m_L)}} = \overline{g^{(m_S)}}$, absent uncertainty the discount premium equals the average yield premium, $\delta^{(m_L)} - \delta^{(m_S)} = \overline{r^{(m_L)}} - \overline{r^{(m_S)}}$.[8] Again referring to Table 1, we find $\delta^{(5)} - \delta^{(.25)} = 0.0108$. As a comparison, note this is the same order of magnitude as the short-term real rate, $\bar{r}^{(.25)} - \bar{\pi} = 0.0177$.

Maturity in years - *m*	$\bar{r}^{(m)}$	$var\left(g^{(m)}m\right)$	$var\left(\pi^{(m)}m\right)$	$cov\left(g^{(m)},\pi^{(m)}m\right)$
0.25	5.36×10^{-2}	2.04×10^{-5}	5.29×10^{-5}	-1.04×10^{-5}
5	6.44×10^{-2}	9.92×10^{-4}	1.05×10^{-2}	-5.33×10^{-4}
—	$\bar{g} = 0.0218$		$\bar{\pi} = 0.036$	
Note: Quarterly data for the period 1954 through 2002. The data are real per capita consumption of nondurables and services, a price index which is the weighted average of the consumption nondurables price index and the consumption services price index, the one-quarter treasury bill rate, and the one through five-year CRSP zero coupon rates. Means are at annual rates.				

Table 1

One might expect these calculations to be substantially changed by the inclusion of uncertainty. The convention, after all, is to assume exponential discounting and explain yield premia by risk. This is not the case. Using the values from Table 1 we plot $\sigma^2_{(m)}(\alpha) =$

[7] Quoting Kocherlakota (1996, page 50): "Note that the risk free rate puzzle comes from the equity premium puzzle: there is a risk free rate puzzle only if α is required to be larger than one so as to match up with the high equity premium".

[8] Temporarily ignoring uncertainty is useful for a benchmark, but see footnote **Error! Bookmark not defined.** for limitations.

$\alpha^2 var(g^{(m)}m) + var(\pi^{(m)}m) + 2\alpha cov(g^{(m)}m, \pi^{(m)}m)$ in Figure 1.[9] For $\alpha = 3$, the effect of risk is to add 13 basis points to the five-year versus one-quarter yield premium. Our estimate of the departure from exponential discounting is $\delta^{(5)} - \delta^{(.25)} = 0.0121$.[10] Inclusion of uncertainty does very little to our calculation of the discount premium, and the small effect it does have is to increase the estimate. If this seems surprising, note that to the extent log consumption and inflation follow random walks, uncertainty has *no* effect on the discount premium calculation as $\sigma^2_{(m)}$ will be proportional to m which will be exactly cancelled by the m in the denominator in equation (5).

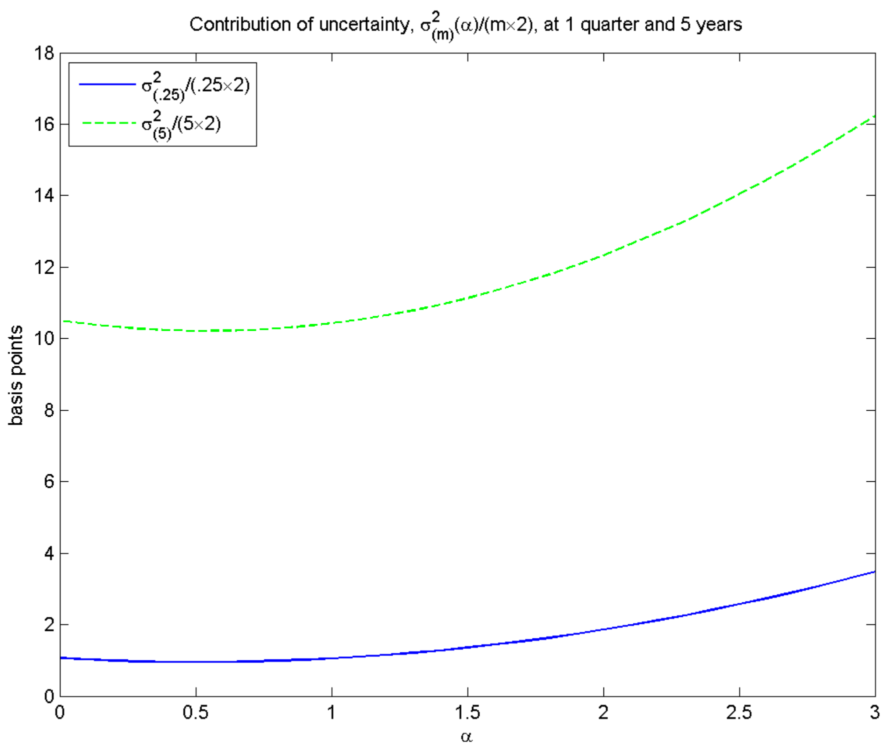

Figure 1

[9] We report unconditional uncertainty measures in Table 1 where one really wants moments conditional on the consumer's information set. It turns out that even unconditional uncertainty is small enough so as not to matter.
[10] Including uncertainty makes equation (4) a quadratic function in α. Solving for the largest admissible α for $m = 0.25$ turns out to give the same bound as before. Including uncertainty, α has to be smaller than .812 (or larger than 534.5, a possibility we ignore).

While it may be that what we are seeing is an additional piece of evidence regarding the equity premium puzzle, these estimates of uncertainty suggest otherwise. Figure 1 shows that for decisions with an *m*-year horizon the risk on of an *m*-year bond is negligible, in fact that the long bond is safer than the short bond. So if one regards the apparent departure from exponential discounting as a puzzle, it looks to be an addition to the existing puzzle list.

Is the departure from exponential discounting economically important? The answer necessarily depends on the application, but one metric is to compare levels of consumption five years in the future that provide equal contributions to utility first assuming a $\delta^{(.25)}$ discount rate and then assuming a $\delta^{(5)}$ discount rate.

In the next section, for $\alpha = 0.5$, we estimate $\delta^{(.25)} = 5.19 \times 10^{-3}$ and $\delta^{(5)} = 1.65 \times 10^{-2}$. The discounted value of utility five years out is $(1+\delta)^{-5} \times (C_{t+5}^{1-\alpha} - 1)/(1-\alpha)$. Call the utility level compensating consumption values with different discount rates $C_{t+5}^{(5)}$ and $C_{t+5}^{(.25)}$. Equation (7) gives the former in terms of the latter.

$$\left(C_{t+5}^{(5)}\right)^{1-\alpha} = 1 + \left(\frac{1+\delta^{(.25)}}{1+\delta^{(5)}}\right)^{-5} \left(\left(C_{t+5}^{(.25)}\right)^{1-\alpha} - 1\right) \tag{7}$$

If we solve Equation (7) setting $C_{t+5}^{(.25)} = 13{,}628$ (mean consumption in our sample), we find $C_{t+5}^{(5)} = 15{,}227$, a 12 percent difference, which we regard as moderately sized. Extrapolating by assuming a flat discount premium outside the range of our data, $\delta^{(30)} = \delta^{(5)}$, we would compute $C_{t+30}^{(5)} = 26{,}538$, just about twice the consumption predicted using exponential discounting based on the one-quarter discount rate. We regard this as a large effect. The difference is smaller at shorter horizons both because the discount rate is small and because of the shorter compounding period. At longer horizons, the compounding effect is greater.

In summary, we find that longer horizon discount rates are relatively much higher than the short horizon rate, but that the absolute level of the discount rate is low. The result is that discounting is of modest importance at short horizons and that the estimated deviation from exponential discounting is of considerable importance at longer horizons. In the next section, we move from calibration to direct estimates of the Euler equations.

Estimation

The left-hand side of equation (2) plus a random error is observable. For a given value of α, the discount rate in the moment condition (8) can be estimated by least squares.[11] Since our yields are not continuously compounded, equation (8) uses powers rather than the mathematically more convenient exponential formulation.

$$E\left[\left(\frac{C_{t+m}}{C_t}\right)^{-\alpha} \times \frac{P_t}{P_{t+m}} \times \left(1 + r_t^{(m)}\right)^m\right] = \left(1 + \delta^{(m)}\right)^m \tag{8}$$

Figure 2 shows estimates of the discount rate as a function of α for the usual one-period Euler equation.[12] Admissible ($\delta > 0$) values of α are somewhat lower than those found from long-run means in the previous section. Using the point estimates, $\alpha = 0.7$ is the highest value for which $\delta > 0$. Values of $\alpha > 1.1$ imply strictly negative confidence intervals for $\delta^{(.25)}$.

[11] In principle, GMM can be used to estimate α and $\delta^{(m)}$ together, but one can get a wide variety of $\alpha's$ depending on the instrument set. See Neely, Roy and Whiteman (2001) and Yogo (2004) for the difficulty of identifying the EIS in the GMM context. Instead, we adopt the strategy of estimating $\delta^{(m)}$ for varying calibrated $\alpha's$. Because we want standard errors to account for serial correlation and to take advantage of contemporaneous correlation across horizons we actually estimate using GMM with Newey-West standard errors. Point estimates are very close to least squares estimates.

[12] Confidence intervals are based on Newey-West standard errors throughout.

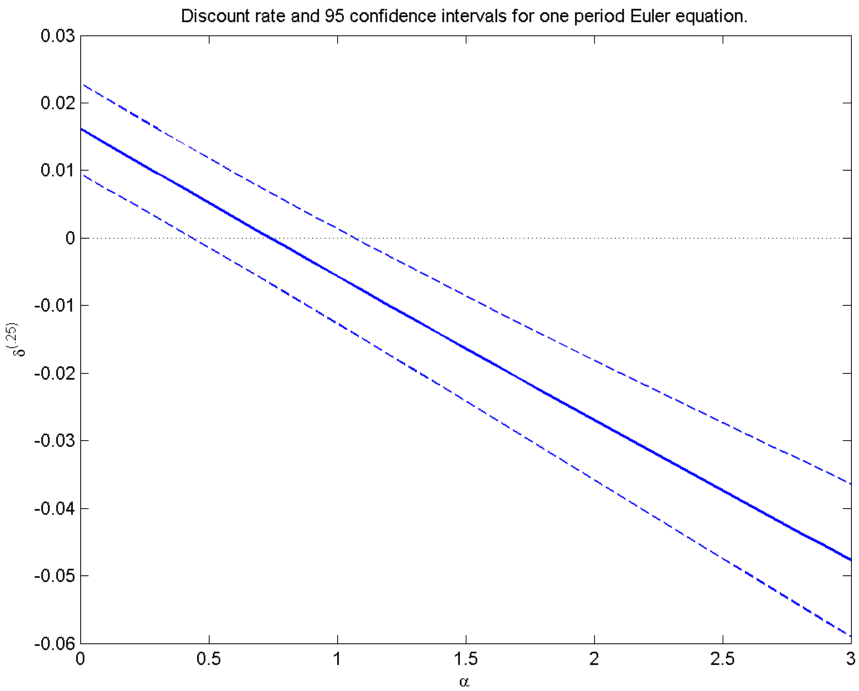

Figure 2

Figure 3 shows the (visually nearly indistinguishable) estimates of the discount premium, $\delta^{(m)} - \delta^{(.25)}$ for $\alpha = 0.5, 1, \text{and } 3$. The premium rises to about 0.011 at five years, which is essentially the same number found in the Introduction. Notably, estimates of the discount premium are unaffected by whether the discount rate itself is positive or negative. Figure 3 also provides confidence intervals, which are somewhat wider at longer maturities and notably wider for successively greater values of α. However, all the estimated discount premia are statistically significant.

The dashed line in Figure 3 shows the mean yield premia. As in the Introduction, we find that discount premia estimates are essentially equal to the yield premia. Uncertainty in consumption and inflation are too small to make much difference.

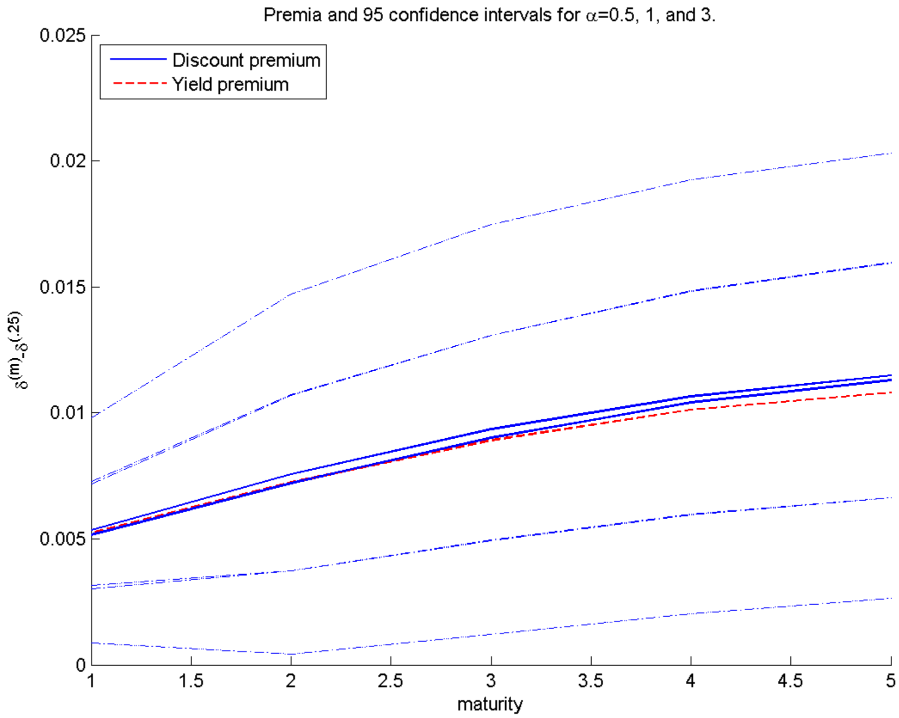

Figure 3

Formally, the null of exponential discounting is $\delta = \delta^{(m)} \forall m$. For values of α in the admissible range, exponential discounting is completely rejected. For $\alpha \leq 1$, the *p*-value is zero to all reported digits. At $\alpha = 2$ the *p*-value is 0.0024. Even at $\alpha = 3$ exponential discounting is rejected, although the rejection is only weakly significant ($p = 0.0875$).[13]

Phelps and Pollak (1968) and Laibson (1997) (see also Angeletos, et. al. (2001)) suggest "quasi-hyperbolic" discounting as a specific departure from exponential. Quasi-hyperbolic discounts take the form $1, bd, bd^2, bd^3$ ... with $d < 1$ and b substantially less than one. However, Some of the important implications of quasi-hyperbolic discounting—Angeletos et. al. (1991) use the term "salience of the present"—depend on b being substantially less than one.

[13] For *much* higher values of α the point estimates of discount premia are considerably larger, but are no longer statistically significant. For $\alpha = 30$, the estimate of the five-year premium is 0.0311.

We fit our six estimated discount rates to $\left(1+\widehat{\delta^{(m)}}\right)^{-m} = bd^m$ by nonlinear least squares. Given the picture in Figure 3 this gives an unsurprisingly near-perfect fit. We find $\hat{b} = 1.006$ with a standard error of 0.002. As we do not find b substantially less than one, our estimates differ from quasi-hyperbolic specifications in an important way.[14]

Further considerations

In this section we look at several further considerations both of interpretation and for empirics.

Figure 3 provides an empirical estimate of discount premia. The associated tests reject the premia equaling zero. We emphasize that one may logically accept the latter while rejecting the former. Our estimates are valid under the null of exponential discounting, which is all the rejection requires. The discount premia estimates require that under the alternative hypothesis the Euler equations follow the usual variational argument at all horizons. But once exponential discounting is abandoned, all sorts of alternative hypotheses can be, and have been, brought forward.

While we would not offer estimates of discount premia if we thought them uninteresting—the Euler equations applying at all horizons is clearly an interesting hypothesis—we cannot overemphasize the care needed in their interpretation. After all, dynamic inconsistency and other deviations in behavior from the canonical model are precisely the reasons that nonexponential discounting is so interesting. But, as an example, consider the finding in the previous section that b is too large to support quasi-hyperbolic discounting. There

[14] Rubinstein (2003) also raises doubts about quasi-hyperbolic discounting, albeit for quite different reasons.

is a logically valid rejoinder that if choices are dynamically inconsistent, then Euler equations do not apply and the apparent deviations of estimated discount schedule from the quasi-hyperbolic are not convincing.

The important point of interpretation being made, we turn to further examination of the empirical estimates.

Longer Horizons

The longer the horizon, the greater the potential interest in non-exponential discounting. Using the CRSP zero coupon data set we are limited to a five year horizon.[15] Robert Bliss has computed an alternative set of zero coupon yields with a much richer set of maturities at both short and long horizons, albeit with shorter historical coverage.[16] Before using the longer Bliss maturities, we compare the common maturities in the Bliss and CRSP data sets for the Bliss period to show that the two are quite similar. Remembering that we estimated $\delta^{(.25)} = 5.19 \times 10^{-3}$ for $\alpha = 0.5$ for the full sample CRSP data, our corresponding estimate using the CRSP data over the Bliss sample period is slightly higher at 6.64×10^{-3}. We find $\delta^{(5)} - \delta^{(.25)} = 1.53 \times 10^{-2}$ for the CRSP data, about 40 basis points higher than for the longer sample period. For the Bliss data we estimate $\delta^{(.25)} = 8.29 \times 10^{-3}$.

Figure 4 puts together CRSP and Bliss-data estimates of the discount premia. The estimates are approximately equal for the one through five-year horizons for which the

[15] While longer term yields are available for coupon bonds, because of the coupons these yields do not exactly correspond to the yields that belong in an Euler equation. Additionally, we discard the last m-quarters of data in order to measure c_{t+m}, which is problematic for large m. With these caveats in mind, note that the yield premium over the five year rate on constant maturity U.S. government bonds from 1977M2 through 2002M2 was 24 basis points for the 10 year rate and 39 basis points for the 30 year rate. (Source: FRED II and authors' calculations.)
[16] See Bliss (1997) and Fama and Bliss(1987). We are grateful to Robert Bliss for making his data available.

datasets overlap. The Bliss-data estimates continue to increase out to the 10-year horizon, increasing with horizon to $\delta^{(10)} - \delta^{(.25)} = 2.59 \times 10^{-2}$. Using the utility compensating measure of consumption in equation (7) gives pretty much the same result as before for the five year horizon. (16 percent using CRSP data and 17 percent using Bliss data, as compared to 12 percent for the CRSP data period.) The difference is much greater at longer horizons. Using the Bliss data a 65 percent increase in consumption is required 10 years out to give equivalent utility. Applying the 10-year discount rate at a 30-year horizon a quadrupling of consumption is required to match the utility flow using the one-quarter discount rate.

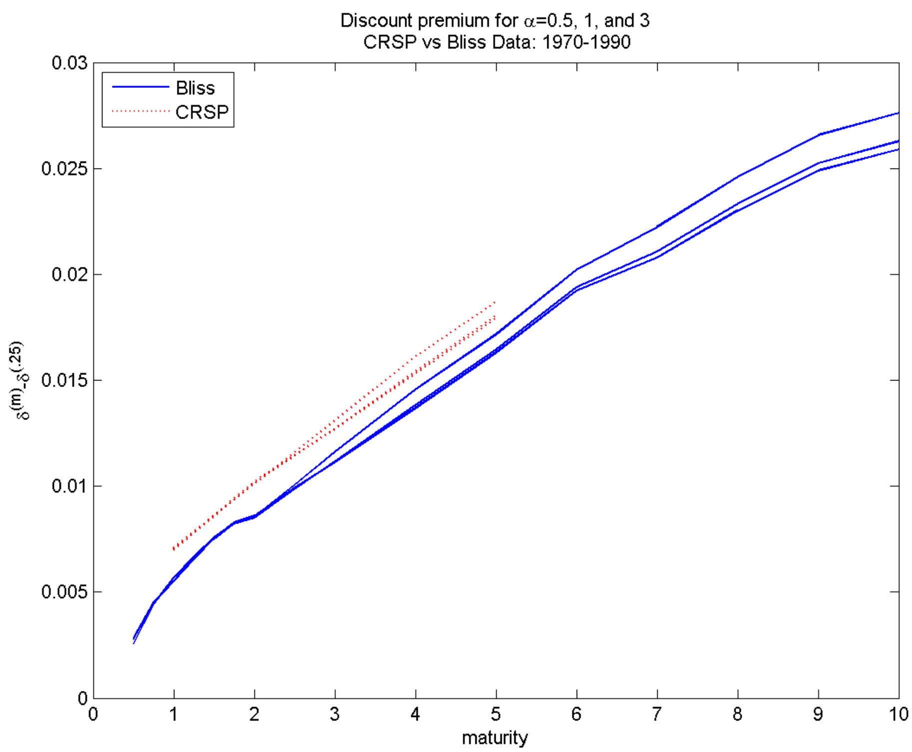

Figure 4

Confidence Intervals

Confidence intervals in Figure 3 are based on Newey-West standard errors. With just under 200 observations there are the equivalent of only 10 non-overlapping observations on five year intervals (loosely speaking). As a check on our Newey-West inference, we compute two parametric bootstraps. First, we estimate the one-quarter Euler equation for $\alpha = 0.5$. We then generate artificial histories for $\frac{C_t^{-\alpha}}{P_t}$ using $\left(\frac{C_{t+.25}^{-\alpha}}{P_{t+.25}}\right) = \left(\frac{C_t^{-\alpha}}{P_t}\right)\left(1 + r_t^{(.25)}\right)^{-.25}\left[\left(1 + \widehat{\delta^{(.25)}}\right)^{.25} + e_t\right]$, where e_t is a residual from the estimated one-quarter Euler equation resampled with replacement. We then re-estimate the system of parameters in Figure 3 one thousand times. This first bootstrap assumes the errors in the estimated one-quarter Euler equation are serially uncorrelated, as theory would suggest[17]. However the empirical residuals are well-modeled as an ARMA(1,1), $e_t = 0.88\mathrm{e}_{\mathrm{t}-1} + \epsilon_{\mathrm{t}} - 0.40\epsilon_{\mathrm{t}-1}$. As a second bootstrap we resample from the estimated innovations, ϵ_{t}, and then generate both errors and the artificial histories.

Figure 5 shows the original confidence region and both simulated $[0.025, 0.975]$ confidence intervals. The simulated intervals are notably tighter than the asymptotic intervals, reinforcing our confidence in the earlier statistical inference.

[17] The Euler equation should be serially uncorrelated if measurement were perfect. Serial correlation may be induced due to both measurement error in the level of consumption and the Working effect. These would induce somewhat offsetting moving average errors. See Wilcox (1992) for a discussion.

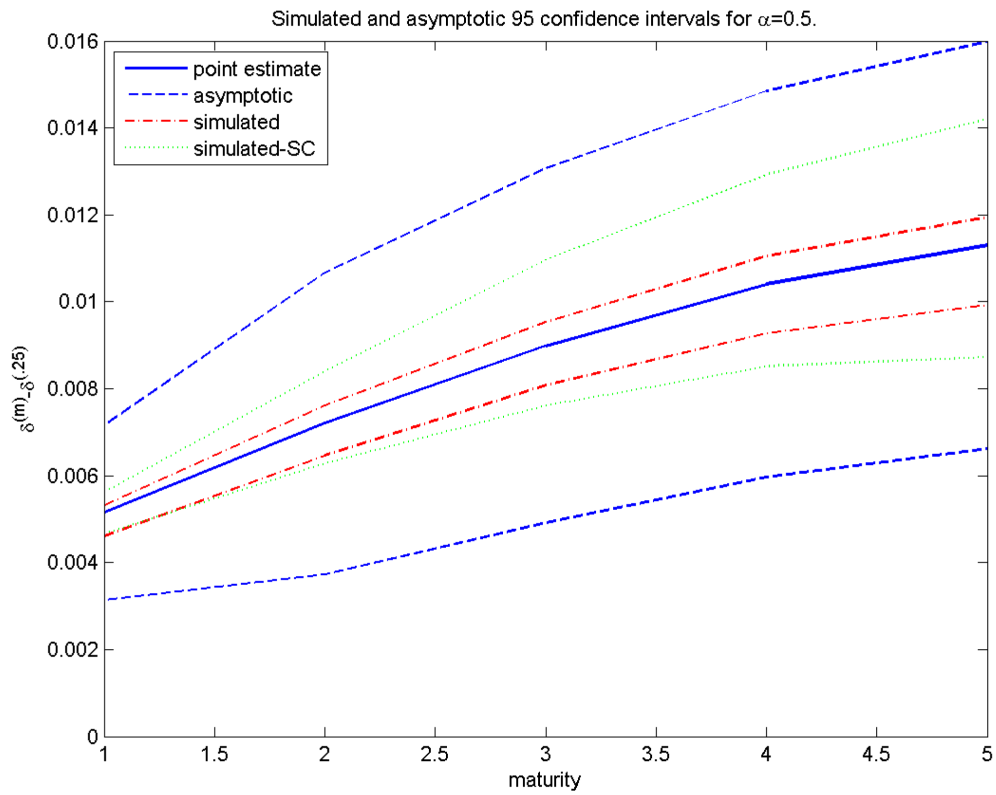

Figure 5

Habit Formation

As an extension to CRRA utility which many writers have found attractive, consider Abel's (1999) external habit formation model. Abel replaces the period utility function with

$$U_t = \frac{1}{1-\alpha}\left(\frac{C_t}{v_t}\right)^{1-\alpha}$$
$$v_t \equiv C_t^{h_0} C_{t-1}^{h_1} \tag{9}$$

where v_t is an external reference standard. Equation (3) would be rewritten as

$$E\left[\left(e^{g_t^{(m)}m}\right)^{-\alpha} \times \left(e^{g_t^{(m)}m}\right)^{-(1-\alpha)h_0} \times \left(e^{g_{t-1}^{(m)}m}\right)^{-(1-\alpha)h_1} \times e^{-\pi^{(m)}m}\right]$$
$$= e^{\left(\delta^{(m)}-r_t^{(m)}\right)m} \tag{10}$$

Taking logs, we see that habit formation adds a lagged variable which in principle could account for serial correlation in equation (3). Equation (4) becomes

$$\delta^{(m)} - r_t^{(m)} = -(\alpha + (1-\alpha)h_0)g_t^{(m)} - (1-\alpha)h_1 g_{t-1}^{(m)} - \pi^{(m)} + \frac{\sigma_{(m)}^2}{2m} \tag{11}$$

External habit formation might account for serial correlation and suggest that the behavioral parameter we identify as α might be $\alpha + (1-\alpha)(h_0 + h_1)$. However, for our long run calculations $\overline{g_t^{(m)}} = \overline{g_{t-1}^{(m)}} = \overline{g^{(m)}}$, so external habit formation ought not affect the estimate of the discount premia. Since for our purposes identification of the behavioral parameters α, h_0, and h_1 is not needed, we modify equation (8) to include lagged consumption growth as in $E\left[\left(\frac{C_{t+m}}{C_t}\right)^{-a_1} \times \left(\frac{C_{t+m-1}}{C_{t-1}}\right)^{-a_2} \times \frac{P_t}{P_{t+m}} \times \left(1 + r_t^{(m)}\right)^m\right] = \left(1 + \delta^{(m)}\right)^m$. Continuing with the strategy of fixing the left-hand side parameters, we estimate this version of the moment condition for $0 \leq a_1 \leq 3$ and $-3 \leq a_2 \leq 3$. Interestingly, for some values of a_2 habit formation improves the results in the sense of eliminating serial correlation and also giving higher estimates of $\delta^{(.25)}$ ("improves" in the sense that the results are closer to our usual priors).

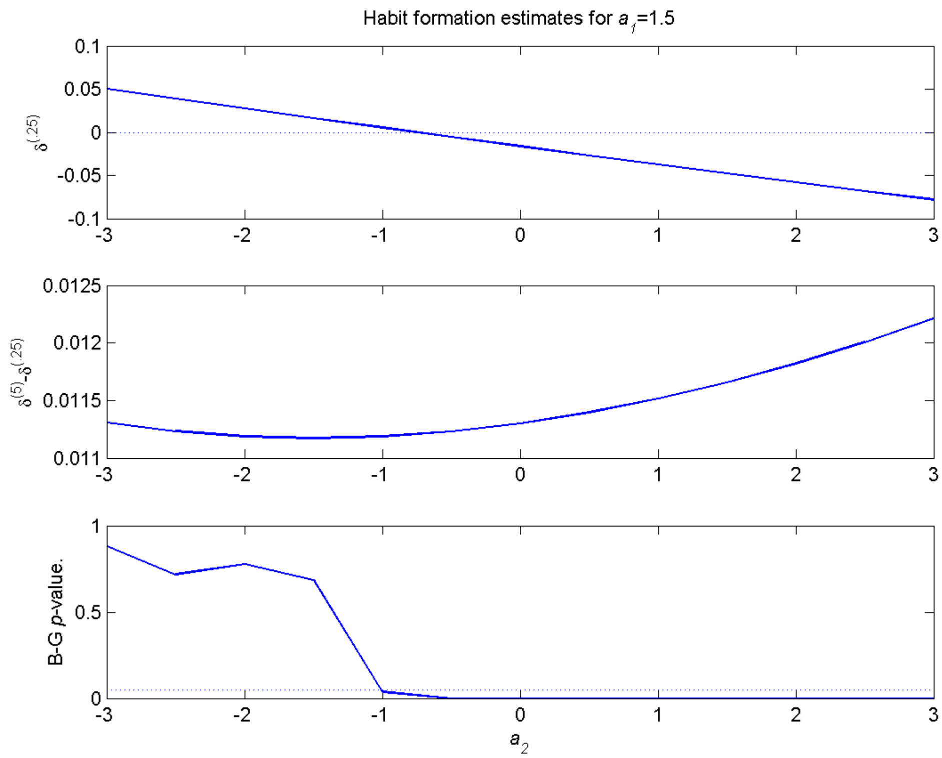

Figure 6

Figure 6 shows habit formation results for $a_1 = 1.5$, which is the value most likely both to give $\delta^{(.25)} > 0$ and to eliminate serial correlation. The $a_2 = 0$ position on the horizontal axis corresponds to the no habit formation results given earlier. The top panel shows, as before, that the estimate of the one-quarter Euler equation gives a negative discount rate and the bottom panel shows there is significant serial correlation. In contrast, for habit formation with $a_2 < -1$ serial correlation disappears and $\delta^{(.25)}$ is positive and generally larger than was estimated without habit formation.

Including habit formation improves the model, but the important conclusion for the purpose at hand is that habit formation makes no difference whatsoever in the estimate of the discount premium which remains between 0.011 and 0.012, as can be seen in the middle panel.

Measurement error

Measurement error in the level of consumption is a potentially important issue in Euler equation estimation (Altonji and Siow (1987) and Singleton (1990)) as it can induce a moving average error in measured consumption that matters more for short-term than for long-term growth rates. From equation (5), one can see that mismeasurement of uncertainty could matter in principle. In practice, uncertainty is too small to for mismeasurement to be an issue. As a check, we conduct a Monte Carlo experiment in which log consumption follows a random walk with mean growth as observed in the data and a shock with one-half the growth variance in the data. We treat this series as simulated "true" consumption. We then add to the simulated *level* of "true" consumption an error scaled to the other half of the variance, giving simulated "noisy" consumption.

We repeat our estimates on one thousand draws of the simulated data. Figure 7 shows median results for both "true" and "noisy" data. The two sets of simulation results are visually indistinguishable from one another (the difference is less than a ten of a basis point, which is smaller than the width of the lines in Figure 7), and as a practical matter are indistinguishable from our actual results. This adds to our confidence that our discount premium estimates essentially reflect differences in yield premia with little effect of uncertainty.

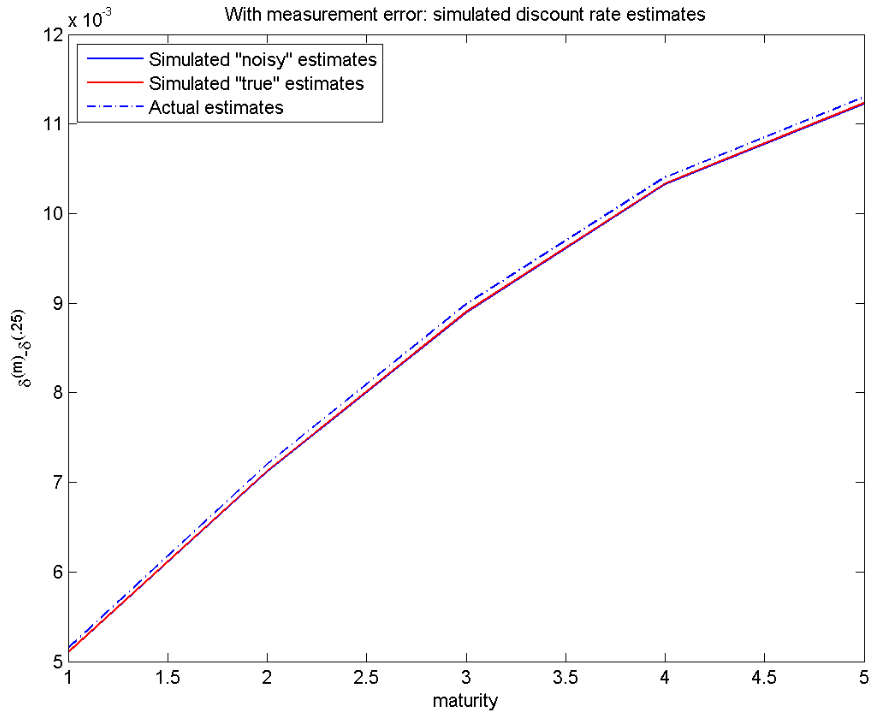

Figure 7

Conclusion

Long-term, safe nominal yields are, on average, higher than short-term yields. In the canonical model of intertemporal expected utility maximization, a positive yield premium implies a positive discount premium—and hence nonexponential discounting—unless the difference is offset by risk. Empirically, uncertainty in consumption paths and inflation is small and we find discount rates rise with the decision horizon.

Exponential discounting is rejected statistically. The economic consequences of nonexponential discounting are modest over short horizons because discount rates are small. Over longer horizons, compounding makes the economic consequences much larger. Looking 30 years out, the difference in utility-equivalent-consumption using a five-year rather than one-quarter discount rate is a factor of two, or if one uses the estimate of a ten-year discount rate from the shorter Bliss data sample, a factor of four.

All this is, of course, a joint test of exponential discounting and the preference specification. Perhaps agents are not infinite horizon, additive expected utility maximizers. Or perhaps aggregation does not give data in which the behavior of the average looks like the behavior of a representative agent.[18] If in addition to rejecting exponential discounting one wishes to accept our estimates of discount premia structure, then one has to accept the use of Euler equations despite the time-inconsistency issues. With that caveat in mind, the departure from exponential discounting is large enough to be critical to an understanding of the differences between short-horizon and long-horizon decision making.

[18] These considerations lead to two suggestions for future research. The first suggestion is consideration of Epstein-Zin (1991) utility to separate risk and time-preference. Doing so would require a way to measure yield premia on total wealth—including human capital investments. As a practical matter, measuring the time pattern on the return to education is problematic. The second suggestion, due to Shelly Lundberg, is to consider whether rising hazard rates for mortality might induce non-exponential discounting without necessarily leading to time inconsistency.

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Appendices – Not For Publication

Data Appendix

Our sample is quarterly from 1954Q1 to 2007Q4.

Nominal yields: The 1-quarter yield data are last-day-of-quarter observations of the 3-Month Treasury Bill (Secondary Market) from the FRED dataset compiled by the St. Louis Fed. The 1 to 5-year zero-coupon yields are last-day-of-quarter observations from the Fama-Bliss Discount Bond Files of the US Treasury database provided by the CRSP. (As an aside, while in principle alternative estimates could be made with real bonds, the history of short-term TIPS is still quite brief. See Gürkayank et. al., Figure 1.)

Real consumption: Data on per capita real consumption on nondurable goods and services are from Table 7, Selected Per Capita Product and Income Series in Current and Chained Dollars, of the National Income and Product Accounts Table compiled by the Bureau of Economic Analysis. The series are in 2000 chained dollars.

Price level: To be consistent with the types of consumption we use, price level is defined by the weighted average of the price indexes of nondurable goods and the price index of service. The two indexes are from Table 1.1.4, Price Indexes for Gross Domestic Product, of the National Income and Product Accounts Table compiled by the Bureau of Economic Analysis. The weight is calculated using the two real consumption series described above. Specifically, on any quarter t, we calculate the weight as $w = \frac{C_{nondurables}}{C_{nondurables}+C_{services}}$, where C_x is the real per capita consumption of type x, and then calculate the price index as $wP_{nondurables} + (1-w)P_{services}$.

Bliss yields: Bliss yields are last month of quarter, 1970-2000, and are supplied courtesy of Prof. Robert Bliss.

Extended Results

Here are point estimates and Newey-West standard errors behind Figure 2. Estimates are by single-equation least squares.

α	$\widehat{\delta^{(.25)}}$	Standard error
0	0.016182	0.003418
0.1	0.013972	0.003389
0.2	0.011768	0.003370
0.3	0.009570	0.003362
0.4	0.007377	0.003364
0.5	0.005189	0.003377
0.6	0.003007	0.003399
0.7	0.000831	0.003431
0.8	-0.001339	0.003472
0.9	-0.003505	0.003523
1	-0.005664	0.003581
1.1	-0.007819	0.003648
1.2	-0.009967	0.003721
1.3	-0.012110	0.003802
1.4	-0.014248	0.003889
1.5	-0.016381	0.003981
1.6	-0.018508	0.004079
1.7	-0.020629	0.004181
1.8	-0.022745	0.004288
1.9	-0.024856	0.004399
2	-0.026962	0.004513
2.1	-0.029062	0.004631
2.2	-0.031157	0.004752
2.3	-0.033246	0.004875
2.4	-0.035330	0.005000
2.5	-0.037409	0.005128
2.6	-0.039483	0.005257
2.7	-0.041551	0.005389
2.8	-0.043614	0.005521
2.9	-0.045672	0.005655
3	-0.047725	0.005790

Figure 3 shows the discount rates and confidence intervals for $\alpha = 0.5, 1, \text{and } 3$. Here are the complete set of discount rates, the *p*-value testing for equality of the discount rates, and the *p*-value for testing the restrictions implicit in quasi-hyperbolic discounting.

α	$\delta^{(.25)}$	$\delta^{(1)}$	$\delta^{(2)}$	$\delta^{(3)}$	$\delta^{(4)}$	$\delta^{(5)}$	*p* (exp discounting)	*p* (quasi-hyper discounting)
0	0.016182	0.021340	0.023388	0.025186	0.026627	0.027531	0	0.0021
0.5	0.005189	0.010336	0.012393	0.014181	0.015598	0.016493	0	0.0249
1.0	-0.005664	-0.000512	0.001563	0.003345	0.004736	0.005624	0	0.2036
1.5	-0.016381	-0.011206	-0.009104	-0.007326	-0.005959	-0.005081	0.0001	0.601
2.0	-0.026962	-0.021749	-0.019611	-0.017835	-0.016492	-0.015623	0.0024	0.7493
2.5	-0.037409	-0.032143	-0.029960	-0.028184	-0.026865	-0.026006	0.0227	0.6382
3.0	-0.047725	-0.042391	-0.040155	-0.038376	-0.037080	-0.036233	0.0875	0.4507

Here are Newey-West standard errors for the difference $\delta^{(m)} - \delta^{(.25)}$

α	$\delta^{(1)} - \delta^{(.25)}$	$\delta^{(2)} - \delta^{(.25)}$	$\delta^{(3)} - \delta^{(.25)}$	$\delta^{(4)} - \delta^{(.25)}$	$\delta^{(5)} - \delta^{(.25)}$
0	0.001154	0.002048	0.002414	0.002616	0.002740
0.5	0.001025	0.001772	0.002080	0.002268	0.002390
1.0	0.001082	0.001790	0.002073	0.002252	0.002369
1.5	0.001291	0.002077	0.002376	0.002557	0.002668
2.0	0.001585	0.002531	0.002880	0.003074	0.003183
2.5	0.001922	0.003069	0.003487	0.003704	0.003815
3.0	0.002276	0.003647	0.004144	0.004389	0.004507

The following are point estimates of the short discount rate from the habit formation model, columns plotted in Figure 6 are highlighted.

$a_2 \backslash a_1$	0	0.5	1	1.5	2	2.5	3
-3	0.085249	0.073465	0.061831	0.050344	0.039003	0.027805	0.016748
-2.5	0.073366	0.061718	0.050219	0.038864	0.027654	0.016585	0.005656
-2	0.061634	0.050121	0.038754	0.027531	0.01645	0.005508	-0.0053
-1.5	0.050052	0.038672	0.027436	0.016342	0.005388	-0.00543	-0.01611
-1	0.038617	0.027368	0.016261	0.005295	-0.00553	-0.01622	-0.02678
-0.5	0.027328	0.016208	0.005228	-0.00561	-0.01632	-0.02688	-0.03732
0	0.016182	0.005189	-0.00566	-0.01638	-0.02696	-0.03741	-0.04773
0.5	0.005177	-0.00569	-0.01642	-0.02701	-0.03747	-0.0478	-0.058
1	-0.00569	-0.01643	-0.02704	-0.03751	-0.04785	-0.05806	-0.06814
1.5	-0.01642	-0.02704	-0.03752	-0.04788	-0.0581	-0.06819	-0.07816
2	-0.02701	-0.03751	-0.04787	-0.05811	-0.06822	-0.0782	-0.08805
2.5	-0.03747	-0.04785	-0.0581	-0.06821	-0.07821	-0.08807	-0.09781
3	-0.04779	-0.05806	-0.06819	-0.07819	-0.08807	-0.09783	-0.10746

The following are Breusch-Godfrey *p*-values from the habit formation model.

$a_2 \backslash a_1$	0	0.5	1	1.5	2	2.5	3
-3	0	0	0.0437	0.8831	0.0511	0.0012	0.0001
-2.5	0	0	0.0538	0.7182	0.0372	0.0019	0.0005
-2	0	0	0.0425	0.776	0.0768	0.0152	0.0125
-1.5	0	0	0.0102	0.6854	0.4747	0.3022	0.3658
-1	0	0	0.0001	0.038	0.2102	0.2872	0.2432
-0.5	0	0	0	0	0.0001	0.0003	0.0005
0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0

The following are point estimates of $\delta^{(m)}$ using the Bliss data.

$m\backslash\alpha$	0	.5	1	1.5	2	2.5	3
.25	0.018754	0.008293	-0.00204	-0.01224	-0.02231	-0.03226	-0.04208
.5	0.021706	0.011152	0.000741	-0.00953	-0.01966	-0.02965	-0.03951
.75	0.023388	0.012845	0.002453	-0.00779	-0.01789	-0.02785	-0.03766
1	0.024335	0.013826	0.003474	-0.00672	-0.01677	-0.02666	-0.0364
1.25	0.025435	0.014891	0.004511	-0.00571	-0.01577	-0.02568	-0.03543
1.5	0.026415	0.015857	0.005466	-0.00476	-0.01483	-0.02474	-0.03449
1.75	0.027179	0.016599	0.006189	-0.00406	-0.01414	-0.02406	-0.03382
2	0.027485	0.016894	0.006474	-0.00378	-0.01386	-0.02379	-0.03355
2.5	0.028755	0.018209	0.007833	-0.00238	-0.01242	-0.0223	-0.03202
3	0.029885	0.019415	0.009112	-0.00103	-0.011	-0.02082	-0.03048
4	0.032342	0.021978	0.011773	0.001723	-0.00817	-0.01792	-0.02752
5	0.034944	0.024593	0.014395	0.004347	-0.00555	-0.01531	-0.02492
6	0.037834	0.02752	0.017355	0.007336	-0.00254	-0.01228	-0.02187
7	0.03929	0.029087	0.019025	0.009104	-0.00068	-0.01033	-0.01985
8	0.041465	0.031311	0.021292	0.011405	0.001648	-0.00798	-0.01748
9	0.043296	0.033192	0.023211	0.013352	0.003613	-0.00601	-0.01551
10	0.044267	0.0342	0.024247	0.014405	0.004675	-0.00495	-0.01446

As a robustness check, we split the sample into 1954-1978 and 1979-2002 subsamples. Subsample statistics are given in Table A 1, along with the analogs of the earlier calculations absent uncertainty. Nothing very much changes, although the effects are larger in the second part of the sample.

subsample	$\bar{r}^{(.25)}$	$\bar{r}^{(5)}$	$\bar{g}$	$\bar{\pi}$	$var(g^{(.25)})$	$var(\pi^{(.25)})$	$cov(g^{(.25)}, \pi^{(.25)})$	max α	$\delta^{(5)} - \delta^{(.25)}$
1954-1978	4.27×10^{-2}	5.11×10^{-2}	0.0250	0.0356	2.25×10^{-5}	6.03×10^{-5}	-1.27×10^{-5}	0.247	0.0084
1979-2002	6.50×10^{-2}	7.83×10^{-2}	0.0184	0.0352	1.67×10^{-5}	4.51×10^{-5}	-8.17×10^{-6}	1.62	0.0133

Table A 1