Are Consumers Forward-Looking?

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Abstract

This paper establishes a stylized fact: in a cross-section of American consumers, consumption is an extremely poor predictor of future income. The PSID now contains the majority of the lifetime income stream for an early cross-section of consumers. Under the assumption that consumption is a function of the expected present value of income, I invert this function and compare the realized present value of income to consumption. Consumption proves to be a very poor predictor of future income, despite future income being predictable by past income. Under rational expectations, information known to the consumer should not enter the regression of present value on consumption. However, as an empirical matter lagged income is a better predictor than is consumption. This is true for both the present value of future income and for income at specific future horizons. Indeed, the relation between consumption and income at increasingly longer horizons weakens quickly. One way to summarize results here is that if you want to forecast individual income and have to choose between consumption and a year-old income datum...go with the latter. The conclusion is that consumers appear "as if" they are not forward-looking and that theories of consumption need to be consistent with this stylized fact. Available data being imperfect much of the paper is devoted to robustness tests, none of which change the basic conclusion.

keywords: consumption, forward-looking behavior, rational expectations, PSID

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1 Introduction

A central assumption in much of modern macroeconomics is that consumers are forward-looking. In particular, consumption choices today are assumed to reflect resources available in the future. This paper establishes a stylized fact: in a cross-section of American consumers, consumption is an extremely poor predictor of future income. Essentially, the PSID has now been running long enough that we can take an early cross-section and then observe realized income over the majority of an individual's earning life. The "stylized fact" is that current consumption contains very, very little information about future income even though future income is predictable (by lagged income). My interpretation is that consumers act "as if " they are not forward-looking. One possibility is that consumers do not look ahead. Alternatively, theories in which consumers do look forward as part of the kind of maximizing behavior we usually assume need to be consistent with the characteristics of the data described in what follows.

More formally, I use what Carroll (1994) calls the "certainty-equivalence (CEQ) LC/PIH model" as an organizing principle. Using a cross section taken from the Panel Study of Income Dynamics (PSID), I calculate the present value of realized income, V_i. Since realized income equals the expected value of income plus a random error term uncorrelated with information available at the time the expectation is taken, $V_i = E(V_i) + \varepsilon_i$. If consumption, C_i, depends on expected future income, $C_i = f(E(V_i))$, then $E(V_i) = f^{-1}(C_i)$, and the relation can be estimated with the obvious, albeit somewhat nontraditional, regression of realized income on consumption, $V_i = f^{-1}(C_i) + \varepsilon_i$, where any variables in the consumer's information set should not enter the regression. A first empirical finding is that this regression not only fails, but fails spectacularly. Since the theory is only an approximation to the solution to a full-blown stochastic dynamic program, it is important that the failure is large so that we can have reasonable faith that the failure is fundamental rather due to the approximation. The second finding is that the relation between consumption and income at increasingly longer horizons weakens quickly.

In looking at the relation between future income and current consumption, I depart from the Euler equation approach that dominates the literature. Attanasio (1999) describes the beauty of the Euler equation approach, saying "The big advantage of the [Euler equation] is the elimination of...the necessity of explicitly modelling the way in which the distribution of future variables influences consumption choices." (page 767.) If the brilliance of the standard Euler approach lies in the assumption that current consumption summarizes forward-looking information, a potential flaw is that this assumption that consumption does summarize information about the future is not directly tested.

The evidence below is based on cross-section regressions of present value of income on consumption and on lagged income. Formal rejection of CEQ follows from showing that lagged income has a large, significant regression coefficient, even though under CEQ it ought not enter. Formal rejection of CEQ is probably of only modest interest, given that there are many other rejections in the literature. This is a case in which R^2 is probably more interesting than coefficient significance; the stylized fact that is established is that lagged income does a much better job of predicting present value than consumption does—just the opposite of what should happen under CEQ. The present value of future income is then broken down into its component summands, $y_{i,t+1}, y_{i,t+2}, \ldots$, and we will see that the failure of consumption as a predictor happens at all horizons. The real surprise is how poorly consumption predicts future income both in terms of a low R^2 on an absolute scale and a low R^2 compared to what results from using lagged income. One way to summarize the result is that if you want to forecast individual income and have to choose between consumption and a year-old income datum...go with the latter.

The substantive sections of the paper are divided in two parts. In the first, I present evidence for the results on the relation between consumption and realized income. In the second, I present a number of robustness tests. Because the available data is less than ideal, these robustness tests are more than *pro forma*. First, some background.

2 Background

2.1 Related literature

The modern empirical literature on consumption as forward-looking behavior derives from Hall (1978). Starting with Flavin (1981), most of the subsequent literature has followed Hall in using an Euler equation approach. (See Attanasio (1999) for discussion and many references.) Hall and Mishkin (1982) applied the Euler equation approach to microdata using the PSID. Flavin (1981) also introduced the idea of "excess sensitivity," with further work in Flavin (1985) and Flavin (1993). The evidence for excess sensitivity is now well established. The results here offer a plausible explanation for excess sensitivity, consumption is overly responsive to current income simply because it is not much at all responsive to expected future income.

A notable exception to the Euler equation approach is Carroll (1994). Carroll creates measures of $E(V)$ by forecasting income based on demographic measures in the Consumer Expenditure Survey (CEX) and then estimating $C = f(E(V))$ by instrumental variables. The CEX has an advantage over the PSID in that it has much broader measures of consumption, but the CEX lacks long horizon measures of income. While Caroll's method uses imputed $E(V)$ rather than using individual income measures of V directly, my intellectual debt is obvious. Carroll finds that "predictable changes in income appear to have no influence on current consumption" (page 112).

Nalewick (2006) combines the Euler equation approach with looking at future income in synthetic panels, finding evidence that changes in consumption have modest predictive power for income growth as distant as six years in the future. In Nalework's framework, this is evidence in support of forward-looking behavior. One reason for the different interpretation of results in this paper is that while I also find modest predictive power of consumption, I show that the predictive power is much less than it should be. I look at longer horizons and use individual rather than synthetic data. On the other hand, Nalewick uses a much richer set of consumption data, an issue dealt with briefly below.

A number of authors, starting with Hall and Mishkin, raise the important issue of measurement error in PSID consumption; see also Zeldes (1989), Runkle (1991), Carroll (1994), and Dynan (2000). Measurement error receives considerable attention below, in particular by use of instrumental variable techniques in the section on robustness checks. Altonji and Siow (1987) consider measurement error in PSID income, although this is less of an issue for what follows here since pure measurement error in the dependent variable is relatively innocuous. Since non-random measurement error can be an issue, I use several different methods of measuring the dependent variable as robustness checks.

Finally, much of what follows involves regressing income on consumption, which is a little unusual, to say the least. So a debt is owed to Fama (1975) for reminding everyone that the choice of which variable goes on the left side of a regression depends on the stochastic specification, and in particular that under rational expectations the choice is not always the expected one.

2.2 Data

Data is from the Panel Study of Income Dynamics, using 1971 as the base year. The unit of observation is the family, as defined by following the head-of-household (PSID variable unqid) through time. All data is in real terms. My dating convention is to date variables according to the survey year in which they are collected. Consumption, C, is measured as the sum of food at home and food away from home, which is the only consumption data collected in the PSID. Income, y_t, equals husband's labor income plus wife's labor income discounted to 1971 at 5 percent. I impute a zero if spousal labor income is missing for one spouse, but not if it is missing for both. Observations of zero labor income are considered legitimate, an issue which is explored later in the paper. The longest contiguous span for which labor income data is available in the PSID is 1970 through 1997. Income variables measure income in the previous year, and are deflated accordingly. While practice varies, I follow Zeldes (1989) in treating consumption as a current year variable. Because the consumption dating convention is arguable, I omit current year income from computation of V, the realized present value of income. V is calculated as $V = \frac{0.05}{1.05}\sum_{t=1972}^{1997} y_t$, where scaling by $\frac{0.05}{1.05}$ provides an innocuous normalization to a rough annuity value. In this way in the regressions of V_i on C_i and of V_i on C_i and y_{70}, V_i is the present value of income reported in 1972 through 1997 for the years 1971 through 1996, C_i is consumption reported in 1971 and presumed to apply to 1971, and y_{70} is income reported in 1970 for 1969 and therefore clearly in the information set at the time C_i is chosen whether one accepts the interpretation that consumption data applies to the reporting year or applies to the preceding year.

As a note, 26 years captures 80 percent of the present value of a 45 year constant annual income working life. In addition to income and consumption, the empirical results use age, measured as age of the head of household in 1971, and the number of members of the household in 1971.

The core sample, which is used except where otherwise indicated, begins with all families for which data is available on consumption and income in all years 1970 through 1997, as well as age and number in the household in 1971. Observations in the highest and lowest 2.5 percentiles of V are then dropped, leaving 831 observations. Descriptive statistics appear in Table 1. Notice that despite trimming the upper and lower tails based on the distribution of V, there remain some high-age observations in which one assumes that most of the working life has been completed, as well as some observations which report curiously low figures for consumption and income. (In fact, there are only five observations with consumption below 1,000 dollars.)

	C	V	y_{1970}	age of head	# in household
mean	$8,061$	$29,602$	$43,444$	38.7	3.68
median	$7,569$	$28,745$	$40,911$	38.0	4.00
maximum	$27,899$	$71,461$	$181,271$	69	14
minimum	844	790	37	19	1
std. deviation	$3,887$	$15,514$	$24,041$	11.2	1.80

Table 1 Core Sample: Descriptive Statistics

3 Consumption and realized income

3.1 Consumption and realized present value.

I begin with estimates of the cross-section relation between consumption and the realized present value of income. The first three regressions in Table 2 provide benchmarks for the relation between V and C. Column (1) shows a simple regression. Not surprisingly the coefficient on consumption is significant, with a t above 7.[1] The low R^2 suggests that either consumption is a very poor predictor, or that future income is very hard to predict.

One might expect the relation between V and C to depend on both age and the number of people living in the household as outcomes of optimal planning, and in the case of age as a result of truncation bias in the measurement of V. Column (2) adds age of the household head and the number in the household. Both variables have large, significant effects and greatly increase predictive power. The third column adds interactions of these variables with consumption; these interactions can be thought of as affecting the marginal propensity to consume. The number in the household has a significant and moderately large interaction effect. The interaction with age is not significant and the point estimate would contribute modestly to explained variation.

Unsurprisingly, consumption is a statistically significant predictor of future income. It is not, however, a very good predictor despite the fact that future income is predictable. Adding lagged income to consumption (column (5) versus column (2) nearly doubles the R^2.

[1] Throughout, standard errors appear in parentheses and one, two, or three stars indicate a coefficient significant at the 10, 5, or 1 percent level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)
constant	21141*** (1197)	47202*** (1826)	40775*** (4028)	41298*** (1479)	41252*** (1474)	35954*** (3233)
consumption	1.05*** (0.134)	1.60*** (0.142)	2.42*** (0.579)		0.319** (0.127)	1.13** (0.468)
age of head		−715*** (40.3)	−726*** (91.2)	−773*** (32.0)	−783*** (32.1)	−694*** (73.0)
# in household		−774** (304)	1091** (526)	301 (199)	−51.3 (243)	423 (422)
age×consumption			0.001 (0.013)			−0.014 (0.010)
# in hh×consumption			−0.208*** (0.048)			−0.056 (0.039)
income_{70}				0.394*** (0.015)	0.374*** (0.017)	0.372*** (0.017)
R^2	0.069	0.326	0.341	0.573	0.576	0.578

Table 2 Regressions on the Realized Present Value of Income

(The R^2 on lagged income alone, omitting age and household size, is 0.27—quadruple the R^2 from having consumption alone on the right.) In fact in column (4), eliminating all terms involving consumption yields an R^2 of 0.573—which is all but identical to the R^2 in columns (5) and (6). So while theory says that variables in the agent's information set should have no predictive power once consumption is accounted for, the fact is that consumption has no predictive power once lagged information is accounted for.

CEQ theory says lagged variables should not enter the regression. As a formal direct test, Column (5) of Table 2 adds lagged income to the regression in column (2). Lagged income, i.e. income in 1970, is strongly significant—the t is 22—adding to the literature's list of refutations of CEQ theory. The coefficient on consumption remains significant, but falls in size by a factor of 5. The sixth column of Table 2 returns the interactions with age and number in household to the regression, giving essentially the same results as in column (5).

3.2 Consumption and Future Realized Income.

By construction, $V = \frac{0.05}{1.05}\sum_{i=1}^{T} y_{t+i}$. In the previous section, we saw that the predictive power of C for V is not as theory suggests. This section examines the relation between C and y_{t+i}. An interpretation of the empirical results which follow is that consumption has a quite small amount of predictive power over a short horizon, that fades completely as the horizon lengthens—despite the fact that longer term income is in fact predictable.

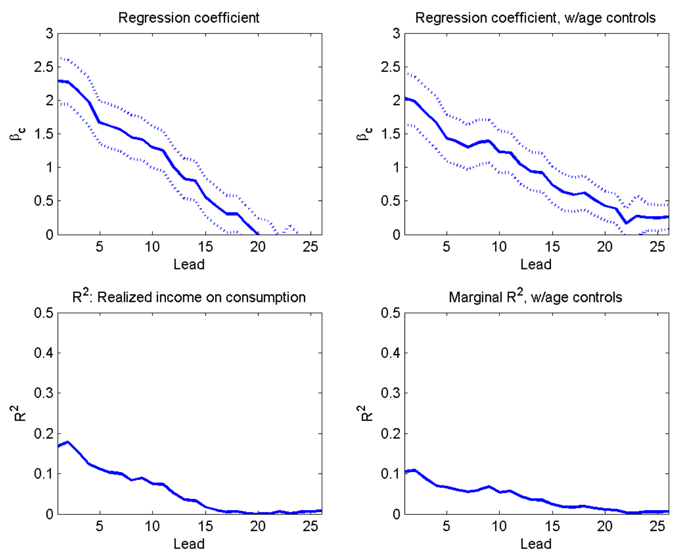

Figure 1

The left panel of Figure 1 shows the estimated coefficient from the regression of y_{t+i} on C.[2] The estimated coefficient remains roughly constant for two years out and then fall linearly, becoming insignificant 19 years out. The associated R^2 is low, never rising as high as 20 percent. From 13 years out, the R^2 never reaches 5 percent and from 15 years out the R^2 never reaches 2 percent. In summary, consumption does a poor job predicting future income.

If consumers have large numbers of zeros for labor income at higher ages, presumably because of retirement, the rapid drop-off in the consumption coefficient will be potentially

[2]Confidence bands are for 95 percent intervals throughout.

misleading when looking at long leads. The right panel of Figure 1 repeats the regressions in the left panel, adding controls for age and age-squared. The regression coefficients drop more slowly and remain statistically significant (except for years 22 and 23). Again, the coefficients are flat for the first two years and then drop linearly. Nonetheless, the predictive power of consumption is very small. The lower right panel shows the marginal R^2 due to consumption (the difference between the R^2 in regressions with consumption and age controls on the right and the R^2 in regressions with only age controls). The marginal R^2 is always below 11 percent, remains below 5 percent from year 12, and below 2 percent from year 16.

These results establish that consumption has disappointingly poor performance at predicting future income across a wide time horizon. Perhaps consumers simply have very little information about future income. However, such is not the case.

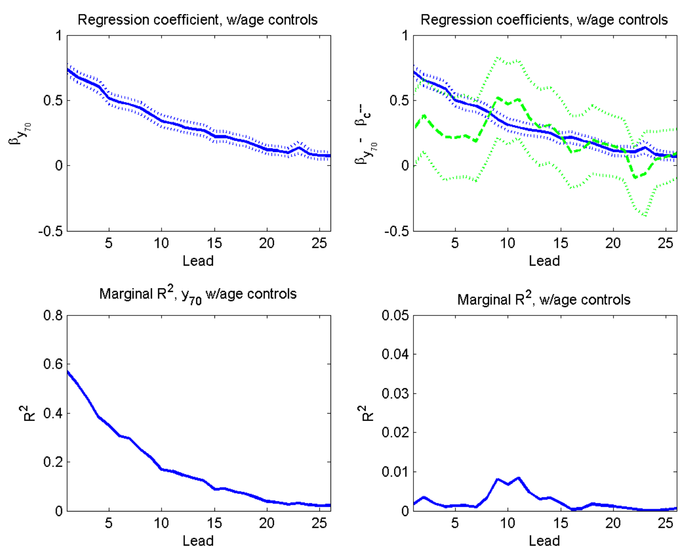

Figure 2

The upper left panel of Figure 2 show estimates parallel to the estimates in the upper right panel of Figure 1, but using lagged income rather than consumption as the predictor (with age and age-squared as controls). As with consumption, the coefficient on lagged income declines linearly, remaining significant at all leads. (The lowest t is above 5.) The fundamental difference in forecasting ability appears in the comparison of the lower left panel in Figure 2 and the lower right panel of Figure 1. Where the marginal R^2 for consumption was below 11 percent at all leads, the same figure for lagged income begins above 55 percent, remains above 10 percent until year 15, and doesn't fall below 5 percent until 20 years out. So the poor predictive power of consumption is not because future income is inherently unpredictable.

The right panel of Figure 2 provides estimates with both consumption and lagged income as predictors. Comparison of the upper right and upper left panels shows that including consumption has essentially no effect on either the point estimates of the lagged income coefficient or on its statistical significance. In contrast, the coefficient estimates for consumption are smaller than in the previous figure.

In fact, consumption has no marginal predictive power at any horizon. As the lower right panel of Figure 2 shows—note the difference in vertical scale between the left and right lower panels—the marginal R^2 for consumption never reaches 2 percent.

In summary, consumption does a poor job of forecasting future income even in the near term future, despite the fact that future income is easily forecastable. The empirical facts suggest, with at most mild exaggeration, that consumption provides no evidence at all of forward-looking behavior.

4 Robustness checks

Perfect data would provide exact observations on consumption and completed labor income histories for a large number of consumers. The PSID data is not perfect. This section begins with robustness checks on four major issues: errors-in-variables in consumption, omitted wealth, truncation of labor income, and the propriety of using lagged income as an element of the consumer's information set. Then I turn to robustness checks that might matter in principle, but that are of less importance in practice.

	(1)	(2)	(3)
constant	15189*** (1640)	45932*** (1882)	40974*** (1483)
consumption	1.18*** (0.165)	2.50*** (0.213)	0.622*** (0.188)
age of head		−764*** (42.1)	−797*** (32.8)
# in household		−1897*** (366)	−412 (293)
income_{70}			0.367*** (0.017)
R^2	0.068	0.294	0.573
$\breve{R}^2$	0.078	0.384	0.579

Table 3 IV Regressions on the Realized Present Value of Income

4.1 Errors-in-Variables in Consumption

Measurement error in the consumption variable would bias the initial regressions. And if CEQ theory held perfectly, measurement error would be expected to bias regressions to look much like the results we see. A downward bias on the consumption coefficient would be offset by upward biases in other parameters. This might, for example, account for the large intercept in column (1) of Table 2 as well as the significant coefficients on lagged income.

Table 3 presents instrumental variable estimates of $f^{-1}()$.[3] A Hausman test comparing the coefficients on consumption in the second columns of tables 2 and 3 gives a $t-$ of 6, supporting the notion that consumption is measured with error. As one might expect when correcting for errors-in-variables, the estimate of the consumption coefficient increases, in this case by more than 50 percent.

The R^2 shown in Table 3 appear to support the notion that consumption is a poor predictor. But this is unfair. The instrumental variable R^2 gives the predictive power of

[3] Instruments for consumption are consumption_{70}, age, age^2, age^3, and number in household, here and in later tables. Other right hand side variables serve as instruments for themselves. The correlation between lagged income and fitted consumption is 0.47.

Note specifically that lagged income does not appear in the instrument list for consumption. This choice of instruments is consistent with the CEQ. Two-stage least squares results are generally even less favorable to the CEQ.

consumption from the econometrician's vantage point, but, assuming the consumer knows her consumption without measurement error, understates the predictive power from the consumer's point of view. If the measurement error is η, then the instrumental variable residuals estimate $\varepsilon - \beta\eta$, while we should only be interested in $\operatorname{var}(\varepsilon)$. We can back out an asymptotically corrected measure of predictive power, $\breve{R}^2$ using

$$\breve{R}^2 = R_{IV}^2 + \beta_{IV}^2 \frac{\sigma_C^2}{\sigma_V^2} \left(1 - R_{z,C}^2\right) \left(1 - \frac{\beta_{ols}}{\beta_{IV}}\right) \tag{1}$$

where σ_C^2 is the variance of measured consumption, σ_V^2 is the variance of the dependent variable, and $R_{z,C}^2$ is the R^2 of measured consumption regressed on the other right hand side variables in the equation. Derivation of $\breve{R}^2$ assumes that the least squares estimate would be consistent except for a classic errors-in-variables problem and that β_{IV} is a consistent estimator.[4] Because β_{IV} is used as an estimate of the true value β in the computation, $\breve{R}^2$ is accurate only if the distribution around β_{IV} is tight. Having announced such caveats, I report $\breve{R}^2$ in circumstances where it should probably be taken with a large grain of salt.

Consistent with the errors-in-variables interpretation, the $\breve{R}^2$ in Table 3 are somewhat larger than the R^2 and slightly larger than the least squares R^2 given in Table 2.

These instrumental variable results suggest while errors-in-variables is present, it does not account for much of the failure of consumption to predict well. Lagged income enters in the third column with a much higher $\breve{R}^2$ than that produced by consumption alone, and with a $t-$ of 21 In summary measurement error is of some importance, but it does not reverse the failure of the consumption to be an adequate predictor.

4.2 Truncated Labor Income

An ideal data set would include the complete history of lifetime labor income for a large number of agents. We have a 26 year window on future income. Several different truncation

[4] Derivation of $\breve{R}^2$ is in an appendix available from the author. A related calculation appears in Hall and Jones (1999).

issues arise. The first issue is that there are a fair number of zeros in the data. Zero is a perfectly valid number to enter in a present value calculation. However, use of a linear regression in the kind of forecasting exercise shown in Figures 1 and 2 is somewhat problematic. The second issue is that for those agents for whom we have captured a relatively small fraction of their working life, we may be omitting substantial assets accumulated from previous labor income.

Figure 3 shows the fraction of the sample reporting zero income at different leads. By 20 years out, a quarter of the sample reports zero income; a figure which rises to 38 percent by the end of the data period. In contrast, for consumers who were 40 or under in 1971 the fraction of zeros is 0.05 and 0.12 respectively. 56 percent of the sample falls in this age range. Consumers 40 and under have relatively few zeros, we observe a large fraction of their working years, and they probably haven't accumulated very large asset holdings.

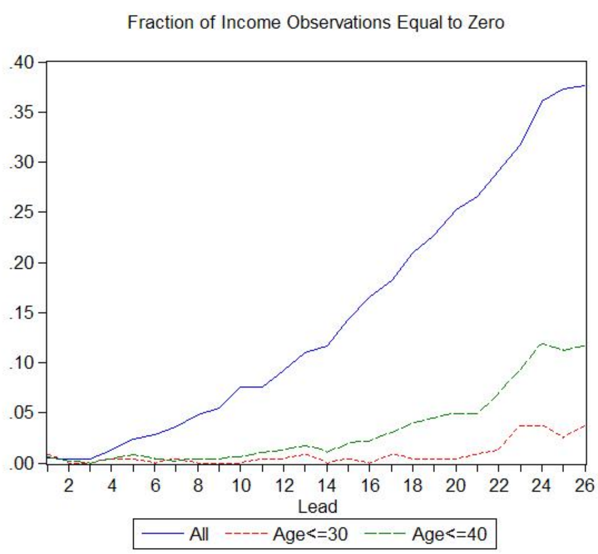

Figure 3

Figure 4 is for this younger subsample (where truncation is not much of an issue), the three panels giving results from left to right consumption, lagged income, and both consumption and lagged income (plus age and age^2 in all cases) to explain income leads. The upper panels show coefficients and 95 percent confidence intervals. The left and middle lower panels give R^2 less the R^2 using age and age^2. The bottom-right panel gives the difference between the R^2 on consumption, lagged income, age and age^2 and the same regressions omitting consumption.

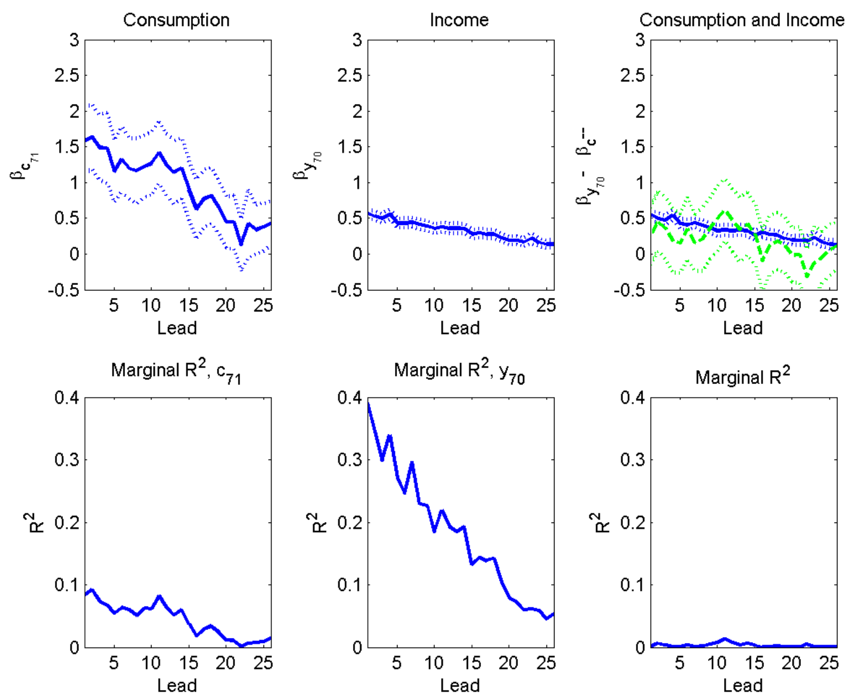

Figure 4

The consumption-only results are visually similar to those in Figure 1. Longer leads remain positive and significant, as one might expect from eliminating most of the zeros. The contribution of consumption to R^2 remains low. The income results in the middle panel are not much changed from Figure 2, so it remains true that the low predictive power of consumption is despite the fact that future income is predictable. Finally, the rightmost panels show, as before, that given lagged income consumption contributes essentially nothing to predicting future income.

Zeros in the income stream do not pose a problem for estimating the relation between consumption and present value. However, large numbers of zeros do suggest that we may have

	(1)	(2)	(3)	(4)	(5)
constant	37983*** (3375)	39745*** (2897)	39905*** (2893)	38853** (3453)	40198*** (2908)
consumption	1.58*** (0.217)		0.364* (0.207)	2.53*** (0.332)	0.739** (0.313)
age of head	−305** (129)	−708*** (115)	−726*** (115)	−448*** (137)	−770*** (118)
# in household	−1633*** (469)	−99.3 (352)	−473 (411)	−2626*** (544)	−864* (479)
income_{70}		0.416*** (0.027)	0.392*** (0.030)		0.384*** (0.030)
R^2	0.104	0.341	0.345	0.067	0.340
$\breve{R}^2$				0.166	0.349
Notes: Columns (1)-(3) estimated by least squares; columns (4) & (5) by instrumental variables, $n = 465$.					

Table 4 Regressions on the Realized Present Value of Income, Consumers Age 40 or Less in 1971

missed an important part of the income stream. Using the younger subsample mitigates this problem while also reducing any issues caused by substantial income from nonlabor sources.

Results parallel to those shown for the full sample appear in Table 4. The R^2 for consumption is actually slightly lower for this younger sample, as is also the case for lagged income. With both consumption and lagged income in the equation, lagged income is highly significant and consumption contributes nothing noticeable to explanatory power. Instrumental variable estimates are consistent with the OLS estimates. Use of the younger subsample confirms findings from the full dataset.

Another approach to adjusting for truncation is to use information on wealth accumulated before the beginning of the observation window. Wealth data early in the PSID is relatively scanty, but there is data on home equity. Table 5 reports estimates in which home equity is added to the realized present value of income.

Including home equity changes no substantive conclusion. Lagged income is highly significant in both OLS and IV estimates. Predictive information comes largely from lagged income rather than consumption.

The 1984 PSID included a supplement asking much more detailed questions about wealth.

	(1)	(2)	(3)	(4)
constant	946292*** (39859)	812432*** (31585)	−714 (5659)	−13676*** (4898)
consumption	37.6*** (3.11)	9.69*** (2.73)	7.61*** (0.639)	2.69*** (0.619)
age of head	−13605*** (880)	−15126*** (689)	591*** (126)	504*** (108)
# in household	−17977*** (7387)	−1722 (5210)	−5114*** (1100)	−1230 (968)
income_{70}		8.42*** (0.364)		0.960*** (0.057)
R^2	0.298	0.575	0.223	0.433
$\breve{R}^2$			0.203	0.432
Notes: Columns (1) & (2) estimated by least squares; columns (3) & (4) by instrumental variables.				

Table 5 Regressions on the Realized Present Value of Income Plus Home Equity

If we are willing to cut the observation window for labor income in half, we can add to the present value of labor income home equity, business ownership, other real estate, IRAs, mutual funds and stocks, the value of vehicles, and other savings, less the value of debt. (I treat missing data for these categories as zeros.) Table 6 gives results.

Recognizing that wealth measures may be quite noisy, nonetheless the results in Table 6 provide no encouragement for theories that require consumption to be a good predictor. In both the least squares and IV estimates R^2 is low and lagged income is highly significant.

4.3 Use of Income As A Lagged Variable

The findings above emphasize not only that consumption provides little information about the expected present value of income, but that this happens despite the fact that a variable representing information readily available to the consumer, lagged income, does have predictive power. Because lagged income is part of lifetime income but outside the calculation window for realized present value of income, in principle the results could reflect an omitted variables problem. This turns out not to be the case.

As an illustration, suppose that our calculation window begins in the consumer's second

	(1)	(2)	(3)	(4)
constant	872253*** (169717)	288015 (178518)	774793*** (173911)	244215 (179402)
consumption	51.2*** (7.57)	27.5*** (7.85)	73.8*** (10.9)	38.3*** (11.3)
age of head	−11355*** (2462)	−4045 (2535)	−11074*** (2478)	−3940 (2537)
# in household	−30735 (22395)	−6882 (21755)	−59630** (24619)	−20605 (24097)
income_{84}		7.55*** (0.931)		7.53*** (0.940)
R^2	0.100	0.167	0.090	0.165
$\breve{R}^2$			0.122	0.172
Notes: Columns (1) & (2) estimated by least squares; columns (3) & (4) by instrumental variables, $n = 820$.				

Table 6 Regressions on the Realized Present Value of Income Plus Wealth in 1984

earning year. In this case, we calculate $V_{t=2}$ as $\sum_{t=2}^{T} y_t$, where the correct calculation is $V_{t=2} + y_{t=1}$. The theoretically correct regression is $V_{t=2} + y_{t=1} = f^{-1}(C_{t=2}) + \varepsilon$. Instead, we estimate $V_{t=2} = f^{-1}(C_{t=2}) + \{\varepsilon - y_{t=1}\}$. It wouldn't be very surprising if lagged income were significant when added to this regression, since it is correlated with the error term.

A first check has already been done in the preceding section when we added wealth measures to the left side of the regression. Including accumulated wealth substitutes for the need to include pre-window income, eliminating pre-window income from the error term and, so eliminating the bias. The fact that including wealth doesn't change the results suggests that use of income as a lagged variable is not an important problem. On the other hand, one might remain suspicious of the quality of the wealth measures.

Consider, then, the bias in β in the regression $V_{t=2} = f^{-1}(C_{t=2}) + \beta y_{t=1} + \{\varepsilon - y_{t=1}\}$. Since ε is the forecast error in lifetime income one expects it to have a large variance. (If this isn't true, then the regression of V on C should have a high R^2.) Therefore, the correlation between lagged income and $\varepsilon - y_{t=1}$ should be small and the bias in the regression should be small. More importantly though, since $y_{t=1}$ and $\varepsilon - y_{t=1}$ are negatively correlated, β should be negatively biased. As empirical estimates of β are robustly positive, the omitted variable problem must not be a significant issue.

Intuition on this and other issues can be aided by an appropriate Monte Carlo. To obtain an approximation to the data generating process under the null, I estimated an AR(2) with fixed effects model for income. The estimated coefficients are then used to simulate 45 years of data for each individual. Realized present value is calculated for a 26 year window. Expected present value is calculated as the future expectation of the AR(2) (under the assumption that individuals know the AR coefficients and their individual fixed effect), starting in the third period, plus realized income in the pre-window period. Consumption is proportional to expected present value. Thus the data generating process assumes that the econometrician's window misses a short period before consumption is measured and a medium length period late in life. Table 7 shows mean results for 1000 simulations. Column (3) confirms the intuition that the omitted variable effect should generate a negative, not positive, coefficient on lagged income. In fact, the estimates are significantly different from zero at the 5 percent level in 63 percent of the simulations.

Columns (4) and (5) of Table 7 give instrumental variable simulations, with the results in column (4) re-estimating the results presented for least squares in column (3). In column (5) I assume that the econometrician's consumption data is equal parts information and noise, the simulated consumption data $\tilde{C}$ being replaced with $\exp\left[\ln\tilde{C}+\nu\right]$, $\nu \sim N\left(0, \mathrm{var}\left(\ln\tilde{C}\right)\right)$. In these simulations the coefficient on lagged income is generally not significant. Thus the instrumental variable results generated under the null are also unlike the empirical findings.

The simulations assume that consumers know only their individual income histories, but do know their own fixed effects, for forming expectations. Thus, the simulation may either under- or over-state the information available to individuals. With this caveat in mind, notice that the simulation results in Table 7 show a relatively high predictive power of consumption (column (1)) and that lagged income adds essentially nothing to predictive power (column (3) versus column (1)). Since empirically the predictive power of consumption is low, and

	(1)	(2)	(3)	(4)	(5)
consumption	23.8 (0.771)		23.8 (0.911)	29.3 (0.907)	6.98 (1.70)
income$_{70}$		10.7 (1.00)	-2.00 (0.87)	-0.372 (0.863)	0.498 (4.04)
R^2	0.531	0.114	0.534	0.532	-7.02
$\breve{R}^2$				0.516	0.477
size nominal 5 percent test $H_0 : \beta_{y70} = 0$		1.00	0.634	0.069	0.0
Notes: Figures in parentheses are standard deviations of Monte Carlo coefficients rather than estimated standard errors. OLS in columns (1)-(3), IV in columns (4)-(5).					

Table 7 Regressions on Simulated Data

essentially adds nothing whatsoever to the predictive power of lagged income, the disparity between simulated and actual results gives some additional evidence that consumption is not forward-looking.

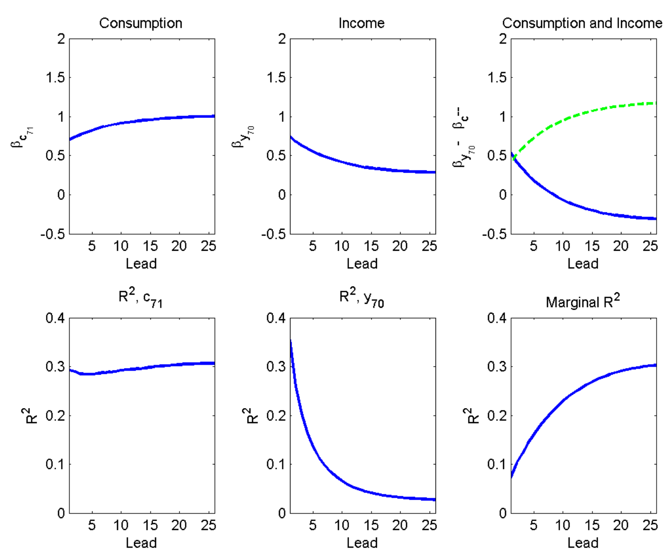

Figure 5

Figure 5 shows simulated results from regressions on leads of income, comparable to those presented in Figures 1 and 2. Simulated R^2 for income is somewhat lower than in the empirical results. In contrast, simulated consumption R^2 and marginal R^2 are both higher than appear in the real data. This provides further, mild, evidence against the forward-looking behavior of consumption.

4.4 Further Robustness Checks

Nothing says that the function $f()$ must be linear. The first two columns of Table 8 show a second-order Taylor series expansion of $f()^{-1}$.[5] The second-order consumption term is significant, but comparison of the R^2 in column (1) to that in the second column of Table 3 shows that allowing for curvature gives a negligible improvement in explanatory power. Lagged income enters in the second column of Table 8 with a $t-$ of 21. So while there is evidence of curvature in $f()^{-1}$, it does not affect the substantive conclusions at all.

Since our measure of consumption is only food, we might expect an income elasticity less than one. Columns (3) and (4) of Table 8 give least squares results regressing $\log V$ on $\log C$. The predictive power of consumption is higher in the log specification than in levels, although of course the $R^2 s$ are not commensurate. The R^2 on age and household size alone is 0.338, so the added predictive power of consumption is not really much higher than it was in levels. However, the addition of log lagged income in column (4) is highly significant and the R^2 rises considerably. In columns (5) and (6) the dependent variable is the average of log realized income instead of the log of the sum of realized income, with periods with zero realized income being omitted from the averaged log. Here, the R^2 on age and household size alone is 0.168. The results are essentially the same as in the middle two columns. Changing the functional form to use logs does not effect the conclusion that the explanatory power of consumption is low and that lagged income is highly significant.

Estimates above assume a five percent discount rate. Table 9 shows results based on no discounting (with non-annuatized V) and on 10 percent discounting, demonstrating that the assumed discount rate has no effect on our conclusions.

A perfect income measure would use labor income adjusted for taxes and transfers. Such

[5] A third-order consumption term added to column (1) was not significant. A second-order lagged income term added to column (2) was significant, and the second-order consumption term became insignificant.

	(1)	(2)	(3)	(4)	(5)	(6)
dependent variable	V	V	$\log V$	$\log V$	$\overline{\log y}$	$\overline{\log y}$
constant	41547*** (2208)	38705*** (1771)	6.36*** (0.393)	4.06*** (0.330)	6.02*** (0.422)	3.88*** (0.380)
consumption	3.23*** (0.393)	1.10*** (0.329)				
consumption^2	-7.73×10^{-5}*** (1.74×10^{-5})	-3.61×10^{-5}** (1.40×10^{-5})				
log consumption			0.612*** (0.048)	0.232*** (0.042)	0.580*** (0.052)	0.227*** (0.048)
age of head	−728*** (40.0)	−788*** (32.1)	−0.042*** (0.002)	−0.043*** (0.001)	−0.029*** (0.002)	−0.030*** (0.002)
# in household	−988*** (304)	−163 (246)	−0.021 (0.014)	−0.001 (0.011)	−0.043*** (0.015)	−0.024* (0.013)
income$_{70}$		0.368*** (0.017)				
log income$_{70}$				0.537*** (0.025)		0.498*** (0.028)
R^2	.342	0.580	0.447	0.649	0.278	0.475

Table 8 Regressions on the Realized Present Value of Income–Alternative Functional Forms

	(1)	(2)	(3)	(4)	(5)	(6)
	discount rate 0 percent			discount rate 10 percent		
constant	2071291*** (72069)	1872393*** (62778)	1870793*** (62662)	49826*** (2198)	41988*** (1666)	41930*** (1659)
consumption	54.4*** (5.62)		11.1** (5.41)	2.11*** (0.172)		0.408*** (0.143)
age of head	−33809*** (1591)	−35741*** (1358)	−36087*** (1366)	−685*** (48.5)	−762*** (36.0)	−775*** (36.2)
# in household	−30850** (11983)	5730 (8459)	−6504 (10337)	−964*** (366)	445** (224)	−4.84 (274)
income$_{70}$		14.0*** (0.675)	13.2*** (0.757)		0.498*** (0.0163)	0.474*** (0.018)
R^2	0.375	0.541	0.544	0.282	0.601	0.605

Table 9 Regressions on the Realized Present Value of Income–Alternative Discount Rates

	(1)	(2)	(3)	(4)	(5)	(6)
	pre-tax income			after-tax income		
constant	38633*** (2375)	36493*** (1901)	36001*** (1893)	28423*** (2198)	27115*** (1596)	26745*** (1593)
consumption	2.19*** (0.187)		0.584** (0.166)	1.57*** (0.172)		0.428*** (0.139)
age of head	−367*** (52.4)	−590*** (43.2)	−599*** (43.0)	−268*** (48.5)	−445*** (36.7)	−450*** (36.5)
# in household	−651* (395)	893*** (260)	249 (317)	137 (366)	982*** (221)	531** (264)
income_{70}		0.460*** (0.018)	0.429*** (0.020)		0.427*** (0.019)	0.397*** (0.022)
R^2	0.197	0.485	0.493	0.195	0.425	0.431
	$n = 824$			$n = 814$		

Table 10 Regressions on the Realized Present Value of Income–Alternative Income Definitions

a measure isn't available, but the PSID does provide measures of total household income (including nonlabor income and transfers) on a pre-tax basis and on an after-tax basis, the latter making an adjustment for estimated federal incomes taxes and FICA. Table 10 provides estimates of the basic regressions using both measures. Nothing of essence changes. Compared to labor income in Table 2, predictive power here is somewhat lower. As before, consumption adds nothing to the predictive power of lagged income and the lagged income variable is highly significant.

The "failure to look ahead" found in the data does not really distinguish between myopia and constraints on acting on what the agent sees. Flavin (1985), using macro data, finds that the excess sensitivity of consumption to current income results from liquidity constraints rather than myopia. Similarly, Zeldes (1989) finds evidence in the PSID for liquidity constraints by looking at Euler equations for different income to wealth classes. Under the assumption that liquidity constraints matter less for wealthier consumers, an indirect test for liquidity constraints can be made by re-estimating the CEQ for the lowest, middle, and highest third of consumers, ranked by lagged consumption, C_{70}.

The results, given in Table 11, look essentially the same for all terciles.[6] If liquidity

[6] Results by tercile restricting the sample to the 40 and under subsample are essentially the same as those

	(1)	(2)	(3)	(4)	(5)	(6)
	lower third		middle third		upper third	
constant	47686*** (2667)	41892*** (2301)	51756*** (3776)	40612*** (3015)	60400*** (5417)	50440*** (4415)
consumption	1.418*** (0.363)	0.391 (0.319)	1.01*** (0.166)	0.202 (0.172)	1.09*** (0.241)	0.138 (0.208)
age of head	−756*** (49.0)	−762*** (41.1)	−686*** (74.2)	−785*** (57.5)	−796*** (103)	−912*** (83.5)
# in household	−821 (557)	−670 (475)	−1001* (539)	−298 (418)	−1235** (510)	73.8 (422)
income_{70}		0.345*** (0.032)		0.438*** (0.032)		0.344*** (0.028)
R^2	0.487	0.640	0.241	0.553	0.214	0.497
R^2 omitting age and household size	0.039	0.186	0.001	0.246	0.041	0.270

Table 11 Regressions on the Realized Present Value of Income–Terciles of Lagged Consumption Level

constraints were important, we would expect to find a greater role for consumption for the bottom tercile than for the top tercile. In fact though, consumption is insignificant while lagged income has a $t-$ greater than 10 in all three subsets. Similarly, in all three terciles lagged income adds to predictive power while consumption does not. So this indirect test doesn't support the idea that the failure of the CEQ is due to liquidity constraints.

Finally, would a broader measure of consumption give different results than are found using the PSID's food consumption measure? We might be concerned about this for two reasons. The first problem is a concern for Engel's law. If the proportion of income spent on food declines so fast that the marginal propensity to consume on food is zero above some income level, then the failure of consumption to predict income might simply reflect that nonresponsiveness of consumption to income level over a relevant range of the data. The second problem is that the behavior of total consumption might be so different from the behavior of food consumption that results using the latter are irrelevant.

The only way to give a really satisfactory answer to such questions is to conduct a similar study with a broader consumption measure. Since this will not be feasible until there are

reported here, although with lower R^2.

representative, longitudinal data sets that include both long income horizons and broad consumption measures,[7] I turn to data from a cross-section of the CEX (Bureau of Labor Statistics, 2003) for a brief comparison of food expenditure to total expenditure. I use 2,780 observations from the first quarter of 1980 to measure food expenditure, total expenditure, and after tax income for a cross-section of families.[8]

How important is Engel's law for the purpose at hand? The evidence in Table 11 above showing similar results for all income terciles already strongly suggested that this is not an important issue. Further evidence comes from the CEX. Figure 6 plots nearest neighbor (Loess) fits on log scales for both food and total expenditure against after tax income. Figure 6 shows that food expenditure remains responsive to income over a broad range. Similarly log ols regressions of consumption on income give elasticities of 0.45 and 0.55 for food and total expenditure respectively, suggesting that the income elasticity of food consumption is not so low as to be a problem.[9]

The cross-section fits in Figure 6 suggest that the difference in the response of food versus total expenditure to income is not great enough to cause an important problem for the purpose at hand. For an element of evidence on the similarity of time series behavior of food versus total consumption, we can compare the CEX reports of expenditure for the current quarter to the reports from the previous quarter. For food the correlation across quarters is 0.45, as compared to 0.35 for the total expenditures. Food consumption is more persistent than is total expenditure, and since persistence is a prediction of the CEQ, this aspect of total expenditure is even further away from the CEQ.

While repeating the analysis with a broader consumption measure would be valuable,

[7] A synthetic panel might be used as an alternative to longitudinal data, as in Nalewick (2006).

[8] I chose 1980Q1 because it is the survey date closest to the PSID consumption date (1971) for which data is conveniently available. The survey universe is reduced first to those families reporting positive food and total expenditure in 1980Q1 and 1979Q4, as well as positive after tax income. I then further reduce the sample by dropping observations in the highest and lowest 2.5 percentiles of the ratios of food expenditure to after tax income and of total expenditure to after tax income.

[9] Note that the independent variable is current rather than permanent income. For the purpose of pinning down an elasticity the latter would be preferable–if consumers were forward-looking. The important element here is only that the two estimated elasticities are roughly the same.

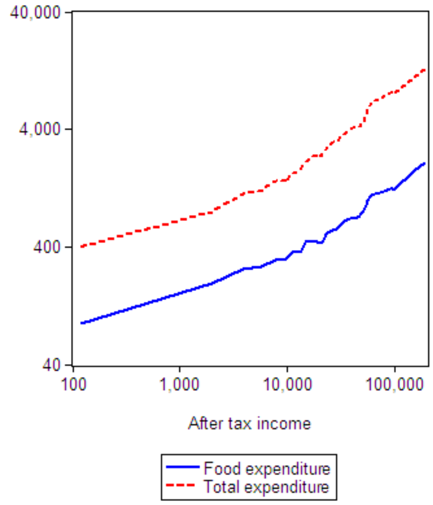

Figure 6

evidence from the CEX does not suggest that greatly different results should be expected.

5 Uncertainty and Precautionary Consumption

Certainty equivalence theory focuses on the first moment of the present value of consumption, but there is ample evidence that higher moments matter as well, presumably for precautionary reasons. (See Carroll (2001), Gourinchas and Parker (2002), and Ludvigson and Paxson (2001) as examples.). Might the apparent failure to look ahead reflect a gross failure of CEQ, but nonetheless be consistent with a forward-looking model that accounts for agents' uncertainty about future income flows? Specifically, might omission of an uncertainty measure account for the poor predictive performance of the level of consumption and the statistical significance of lagged income? While a complete answer requires both a complete model and a measure of agents' *ex ante* income uncertainty, a reasonable robustness test is possible by adding a proxy for uncertainty to $f()^{-1}$. I use two such proxies here. The evidence is consistent with the inclusion of uncertainty in $f()^{-1}$, but does not change the fundamental conclusions in the paper.

	(1)	(2)	(3)	(4)
constant	40328*** (1609)	41527*** (1410)	40331*** (1616)	41405*** (1418)
consumption	0.301** (0.128)	0.208* (0.122)	0.633*** (0.191)	0.539*** (0.200)
age of head	−781*** (32)	−823*** (31)	−795*** (33)	−840*** (32)
# in household	0.766 (246)	45 (233)	−397 (299)	−325 (293)
ARCH std. dev.	0.103 (0.072)		0.074 (0.073)	
unconditional std. dev.		0.628*** (0.071)		0.589*** (0.074)
income_{70}	0.371*** (0.017)	0.202*** (0.025)	0.362*** (0.017)	0.195*** (0.026)
R^2	0.577	0.612	0.573	0.609
$\breve{R}^2$			0.581	0.616
Notes: Columns (1) & (2) estimated by least squares; columns (3) & (4) by instrumental variables.				

Table 12 Regressions on the Realized Present Value Including Measures of Income Uncertainty

The first proxy estimates an AR(1) with fixed effects model for income, and then estimates an ARCH(1) model on the residuals.[10] For each agent, uncertainty is then measured as the square root of the variance predicted from the ARCH(1) model using the 1971 residual. This estimate effectively assumes the agents know no more than the econometrician. The second proxy uses the unconditional standard deviation of income for each individual, which assumes that agents knew future volatility exactly. Table 12 gives results using both measures.

The first two columns in Table 12 are comparable to column (4) of Table 2. The ARCH proxy is insignificant and does not change predictive power. In contrast, the unconditional standard deviation is very significant and modestly increases predictive power,[11] although

[10] As a side note, AR(2) and ARCH(2) coefficients are both significant. Because only one lag of income is available, I use the lower order model. The difference in explanatory power in the AR/ARCH estimation is not large.

[11] It is worth noting that what's going on on a mechanical level is that the unconditional standard deviation is picking up a scale effect while the ARCH based measure is not. The correlation between lagged income and the unconditional standard deviation is 0.82, while the correlation between lagged income and the ARCH based measure is only 0.19.

the coefficient on consumption is only weakly significant. However, the significance of lagged income is unchanged. Instrumental variable results in the right two columns yield similar conclusions.

The foresaid notwithstanding, the estimates including the unconditional standard deviation are different in one important way: the unconditional standard deviation is a good predictor of future income. Thus there is prima facie evidence of forward-looking behavior acting through a precautionary motive. The difficulty is that one has to argue that the precautionary motive is so strong as to completely dominate the expectations motive. This seems particularly implausible in a cross-section. Figure 7 shows the situation.

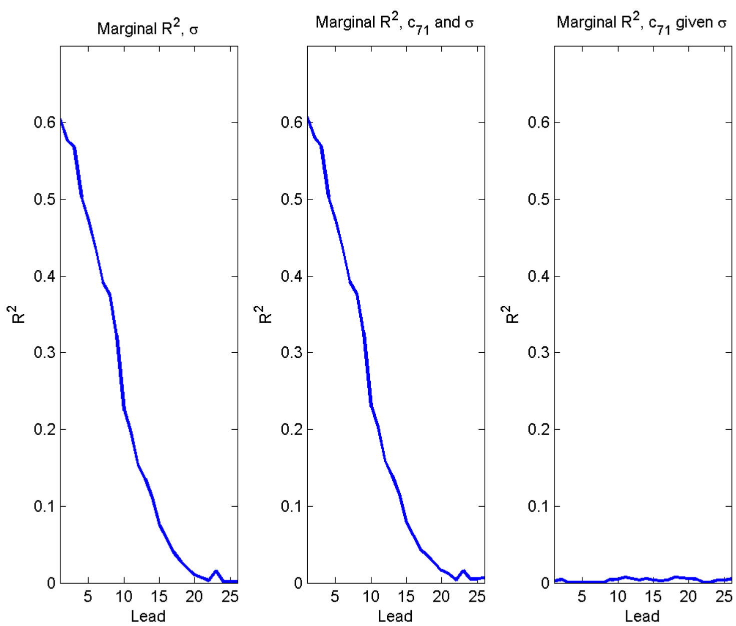

Figure 7

The left panel of Figure 7 gives the predictive power (in excess of the contribution of age, as above) of the unconditional standard deviation. The predictive power is high, especially at early leads. In fact, the predictive power of the unconditional standard deviation is somewhat higher than the predictive power of lagged income until 13 years out, after which lagged income gives slightly better predictions. The middle panel shows the joint predictive power of consumption and the unconditional standard deviation. The right panel confirms that the improved predictive power comes entirely from the unconditional standard deviation, leaving no role at all for consumption. Thus the data supports previous findings that uncertainty is very important, but does not support the idea that consumption reflects information about future income.

6 Concluding Remarks

In making consumption decisions, consumers do not much look forward to expected future income, or at least they act as if they don't. This is despite the fact that future income is predictable. The certainty-equivalence version of the LC/PIH is not the most sophisticated model of individual optimizing behavior. On the other hand, departures from the simple CEQ is very large. It is may be a challenging task to build a forward-looking, optimizing model that accords with the stylized facts presented here.

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7 Appendix (Not for publication)

This appendix derives equation (1). W.o.l.g., let y be the LHS variable, x the true RHS variable, x^* be the RHS variable measured with error, and z represent other RHS variables.

Let the data generating process be

$$y = \beta_1 x + \beta_2 z + u$$

where you observe $x^* = x + \varepsilon$. Our interest is to obtain an estimate $\breve{R}^2 \approx 1 - \frac{\sigma_u^2}{\sigma_y^2}$

By using IV, you get an estimate of the true β and this leads to IV residuals

$$e = y - \beta_{IV} x^* - \beta_{2,IV} z = y - \beta_{IV}(x + \varepsilon) - \beta_2 z \approx u - \beta\varepsilon$$

We need to clean out the term $\beta\varepsilon$. We begin by finding the variance of x, which leads to the variance of ε. Derive σ_x^2 by first getting β_{ols}. Assume plims are taken where useful.

$$\begin{bmatrix} \beta_{1,ols} \\ \beta_{2,ols} \end{bmatrix} = \begin{bmatrix} \sigma_{x^*}^2 & \sigma_{x,z} \\ \sigma_{x,z} & \sigma_z^2 \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \sigma_x^2 + \beta_2 \sigma_{x,z} \\ \beta_1 \sigma_{x,z} + \beta_2 \sigma_z^2 \end{bmatrix}$$

where we use $\operatorname{cov}(x + \varepsilon, z) = \operatorname{cov}(x, z)$.

$$\begin{bmatrix} \beta_{1,ols} \\ \beta_{2,ols} \end{bmatrix} = \frac{1}{\sigma_{x^*}^2 \sigma_z^2 - \sigma_{x,z}^2} \begin{bmatrix} \sigma_z^2 & -\sigma_{x,z} \\ -\sigma_{x,z} & \sigma_{x^*}^2 \end{bmatrix} \begin{bmatrix} \beta_1 \sigma_x^2 + \beta_2 \sigma_{x,z} \\ \beta_1 \sigma_{x,z} + \beta_2 \sigma_z^2 \end{bmatrix}$$

$$\begin{aligned} \beta_{1,ols} &= \frac{\beta_1 \sigma_x^2 \sigma_z^2 + \beta_2 \sigma_{x,z} \sigma_z^2 - \beta_1 \sigma_{x,z}^2 - \beta_2 \sigma_z^2 \sigma_{x,z}}{\sigma_{x^*}^2 \sigma_z^2 - \sigma_{x,z}^2} \\ \beta_{1,ols} &= \frac{\beta_1 \sigma_x^2 \sigma_z^2 - \beta_1 \sigma_{x,z}^2}{\sigma_{x^*}^2 \sigma_z^2 - \sigma_{x,z}^2} \\ \beta_{1,ols} &= \beta_1 \frac{\frac{\sigma_x^2}{\sigma_{x^*}^2} - \rho_{z,x^*}^2}{1 - \rho_{z,x^*}^2} \end{aligned}$$

So using $\beta_{1,iv}$ to estimate β_1 and R^2_{z,x^*} to approximate ρ^2_{z,x^*},

$$\sigma_x^2 = \sigma_{x^*}^2 \left[\frac{\beta_{1,ols}}{\beta_{1,iv}} \left(1 - R^2_{z,x^*}\right) + R^2_{z,x^*} \right]$$

$$\sigma_\varepsilon^2 = \sigma_{x^*}^2 - \sigma_x^2 = \sigma_{x^*}^2 - \sigma_{x^*}^2 \left[\frac{\beta_{1,ols}}{\beta_{1,iv}} \left(1 - R^2_{z,x^*}\right) + R^2_{z,x^*} \right] = \sigma_{x^*}^2 \left(1 - R^2_{z,x^*}\right) \left(1 - \frac{\beta_{1,ols}}{\beta_{1,iv}}\right)$$

$$\begin{aligned}
\sigma_u^2 &\approx \sigma_e^2 - \beta_{1,iv}^2 \sigma_{x^*}^2 \left(1 - R^2_{z,x^*}\right) \left(1 - \frac{\beta_{1,ols}}{\beta_{1,iv}}\right) \\
\frac{\sigma_u^2}{\sigma_y^2} &\approx \frac{\sigma_e^2}{\sigma_y^2} - \beta_{1,iv}^2 \frac{\sigma_{x^*}^2}{\sigma_y^2} \left(1 - R^2_{z,x^*}\right) \left(1 - \frac{\beta_{1,ols}}{\beta_{1,iv}}\right) \\
1 - \frac{\sigma_u^2}{\sigma_y^2} &\approx 1 - \left(\frac{\sigma_e^2}{\sigma_y^2} - \beta_{1,iv}^2 \frac{\sigma_{x^*}^2}{\sigma_y^2} \left(1 - R^2_{z,x^*}\right) \left(1 - \frac{\beta_{1,ols}}{\beta_{1,iv}}\right) \right) \\
\breve{R}^2 &= R^2_{IV} + \beta_{1,iv}^2 \frac{\sigma_{x^*}^2}{\sigma_y^2} \left(1 - R^2_{z,x^*}\right) \left(1 - \frac{\beta_{1,ols}}{\beta_{1,iv}}\right)
\end{aligned}$$