

Growth Differential of Bass Stock

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Executive Summary

This paper will explore the growth differential of a certain species of Bass, from various lakes around the United States, and compare the calculated length-weight coefficient against an allometric model often used in biology to estimate growth. After testing the coefficient against the allometric model, explore the individual characteristics of each lake, and determine which lakes, if any, have significant effect upon the population mean distribution.

Often in biology, growths of animals are categorized into two types: Isometric and Allometric Growth. Isometric growth is defined when an animal grows, and does so without either changing its shape or its density. Therefore, if an animal does not change its shape when it grows, it is defined that its volume is proportional to any linear measure of its size. Its growth equation is defined as:

$$\text{Weight} = a \cdot (\text{Length})^3$$

However, most species of fish do change either their shape or density as they grow, their growth is not isometric. They are categorized to experience allometric growth. Allometric growth is defined as:

$$\text{Weight} = a \cdot \text{Length}^b$$

where the parameter 'b' is approximately 3, but not equal to 3.

The sample of bass under investigation like most other species of fish, undergo allometric growth. Bass do change shape and density, as they grow. Therefore, we expect the results to reflect such growth, where the value of 'b' is close to, but not exactly 3.

Background and Data

In fisheries science, growth of fish is an important ecological question. Growth patterns of any species, can give scientists important details of patterns such as living conditions and environment. If normal growth patterns are present, then there is no cause for concern or further study. However, indications of stunted growth can mean a number of things, such as:

- 1) a warning sign of a greater ecological problem such as pollution
- 2) variation in food supply
- 3) density of fish population per unit volume of water.

Such signs warrant further studies into the specific stock of fish, to possibly determine the cause of growth abnormalities.

The data used to conduct this survey was provided by the National Oceanic Atmospheric Administration (NOAA). There are 5188 overall observations from 7 different lakes around the United States. The observations are as follows:

Weight: weight of bass in grams.

Length: length of bass in millimeters

?: the lake effect upon the sampled bass (also denotes which lake the sampled bass originated).

ln(Weight): log of weight in grams. This is used to transform a non-linear regression to linear form.

ln(Length): log of length in millimeters. This is used to transform a non-linear regression to linear form.

The full model expected to be used is:

$$\ln\text{Weight}_{ij} = a_{ij} + \beta_{1i}(\ln\text{Length})_{ij} + \beta_{2i}?_i + e_{ij}$$

Where a_i is the intercept, and β_{2i} is treated as a dummy variable with a value of either 0 or 1, given the sample stock originates from a specific lake. The value 'e' is the error term.

The variable γ_i depends on the dummy variable β_{2i} , which will allow for more accurate regression models for each lake. The variable γ_i in this case, is known as the “lake effect.” The lake effect, is defined to be the effect each individual lake directly has upon the growth of a fish.

Hypothesis

Allometric growth, as stated above, is where the log weight to log length relationship is approximately 3, but not exactly 3. Therefore, an analysis needs to be performed to determine if the output coefficients from our regressions can be considered isometric or allometric.

$$H_0: \beta_{1i} = 3 \text{ and } H_1: \beta_{1i} \neq 3$$

If any lake shows isometric growth, further research into the possibilities of growth abnormalities should be conducted. Otherwise, the lakes which exhibit allometric growth, are pooled into a separate subset. Using this subset of lakes exhibiting allometric growth, we can test to determine whether any specific lake of the subset, may possibly be significantly different than the population mean.

$$H_0: a = a_1 \text{ } \beta = \beta_1 \text{ and } H_1: a \neq a_1 \text{ } \beta \neq \beta_1$$

If there is, then it can be determined which lakes have the most effect upon the population mean. By using a lake as the reference, we can then graph the difference of the coefficients with their upper and lower limits by ascending rank, and visualize the effect.

Results

Observing the individual regressions for each lake, none of them exhibited any results which were not expected. The R-squared value in all regressions was above 0.90, which shows that there is relatively little variation is explained in this model.

The first series of regressions on each individual lake satisfies our expectation of allometric growth. All of our coefficients of $\ln(\text{length})$ do indeed seem to be close to three, and none of them are exactly 3. How do we know if the coefficient is statistically close to 3, or not close to 3? By performing our first hypothesis as stated above, we use observed the t-statistic. Each coefficient showed enough statistical significance to reject our null hypothesis of the sample stock of bass experiencing isometric growth at a 95% level. This was the expected result. If the sample stock of bass exhibited isometric growth, there would be cause for concern, and further studies into the surrounding conditions of the ecology and life around the lake would be needed.

Now that it is established that each lake does indeed follow our expectations, where the bass exhibit allometric growth, we can observe the growth differential of the lakes.

Using the F-test, we tested to see if there was any possible variation of the coefficients in our full model to our reduced model. In Appendix 3, it shows that there is exists a statistical significance that our coefficients in the full model may be different from our reduced models. Due the fact that such significance exists, we must determine which lakes contain such discrepancies.

Using lake 1024 as the reference lake, and regressed it against the 6 other lakes. Our results show the coefficient of the reference lake to be 3.22, with a standard error of

0.0074. The rest of the output coefficients are displayed by the difference in respect to the reference lake. Running down the calculated critical t-values, we discover that lakes #1, 2, and 5, corresponding to lakes 111, 114, and 755, are statistically significant.

To visualize what this significance is, we can graph the differences by ascending rank as seen in Appendix 3. Since Lake 1024 (#7) was used as a reference it is given a value of zero. It's upper limit and lower limit is very close to, but not exactly the reference standard error of 0.0074. However, it is considered close enough to use as the standard error for Lake 1024 (#7) if we had run a regression using a different lake as reference.

This indicates that the lakes which are statistically insignificant are relatively close to the population mean. Lake 111 (#1), 114 (#2), and 755 (#5) are significant, and therefore have the most affect upon the population mean. Though they exhibit signs of allometric growth, and not isometric growth, the sample bass from these lakes are noteworthy, in that they are significantly different than the population mean. This is not cause for immediate concern, but those lakes are a possible candidate for further studies to determine their possible causes of their difference in growth.

Appendix 1

Dependent Variable: LNWEIGHT

Method: Least Squares

Date: 12/08/02 Time: 18:55

Sample(adjusted): 4650 5188 IF LAKE=1024

Included observations: 539 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-13.39915	0.177335	-75.55862	0.0000
LNLENGTH	3.375492	0.031916	105.7608	0.0000
R-squared	0.954190	Mean dependent var		5.323970
Adjusted R-squared	0.954105	S.D. dependent var		1.120633
S.E. of regression	0.240075	Akaike info criterion		-0.012024
Sum squared resid	30.95063	Schwarz criterion		0.003893
Log likelihood	5.240473	F-statistic		11185.34
Durbin-Watson stat	1.951952	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT

Method: Least Squares

Date: 12/08/02 Time: 18:52

Sample(adjusted): 1 854 IF LAKE=111

Included observations: 854 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.96495	0.105361	-123.0532	0.0000
LNLENGTH	3.308431	0.018669	177.2152	0.0000
R-squared	0.973587	Mean dependent var		5.674687
Adjusted R-squared	0.973556	S.D. dependent var		1.105510
S.E. of regression	0.179773	Akaike info criterion		-0.591908
Sum squared resid	27.53513	Schwarz criterion		-0.580784
Log likelihood	254.7446	F-statistic		31405.23
Durbin-Watson stat	1.293554	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT

Method: Least Squares

Date: 12/08/02 Time: 18:52

Sample(adjusted): 855 969 IF LAKE=114

Included observations: 115 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.16549	0.211679	-57.47140	0.0000
LNLENGTH	3.191097	0.037286	85.58354	0.0000
R-squared	0.984807	Mean dependent var		5.930973
Adjusted R-squared	0.984672	S.D. dependent var		0.856474
S.E. of regression	0.106036	Akaike info criterion		-1.632844
Sum squared resid	1.270523	Schwarz criterion		-1.585106
Log likelihood	95.88851	F-statistic		7324.542
Durbin-Watson stat	1.344654	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT
 Method: Least Squares
 Date: 12/08/02 Time: 18:53
 Sample(adjusted): 970 1913 IF LAKE=304
 Included observations: 944 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.63921	0.096407	-131.1023	0.0000
LNLENGTH	3.244566	0.016991	190.9554	0.0000
R-squared	0.974817	Mean dependent var		5.750049
Adjusted R-squared	0.974790	S.D. dependent var		0.874225
S.E. of regression	0.138806	Akaike info criterion		-1.109360
Sum squared resid	18.14966	Schwarz criterion		-1.099084
Log likelihood	525.6180	F-statistic		36463.97
Durbin-Watson stat	1.864787	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT
 Method: Least Squares
 Date: 12/08/02 Time: 18:54
 Sample(adjusted): 1914 2081 IF LAKE=433
 Included observations: 168 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-12.03108	0.197978	-60.76981	0.0000
LNLENGTH	3.163355	0.035260	89.71397	0.0000
R-squared	0.979792	Mean dependent var		5.701948
Adjusted R-squared	0.979670	S.D. dependent var		1.016577
S.E. of regression	0.144946	Akaike info criterion		-1.013082
Sum squared resid	3.487535	Schwarz criterion		-0.975892
Log likelihood	87.09890	F-statistic		8048.597
Durbin-Watson stat	2.286420	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT
 Method: Least Squares
 Date: 12/08/02 Time: 18:54
 Sample(adjusted): 2082 2603 IF LAKE=755
 Included observations: 522 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-13.30066	0.142783	-93.15303	0.0000
LNLENGTH	3.363939	0.025308	132.9222	0.0000
R-squared	0.971410	Mean dependent var		5.658309
Adjusted R-squared	0.971355	S.D. dependent var		0.885737
S.E. of regression	0.149909	Akaike info criterion		-0.953753
Sum squared resid	11.68581	Schwarz criterion		-0.937440
Log likelihood	250.9295	F-statistic		17668.31
Durbin-Watson stat	1.856559	Prob(F-statistic)		0.000000

Dependent Variable: LNWEIGHT

Method: Least Squares

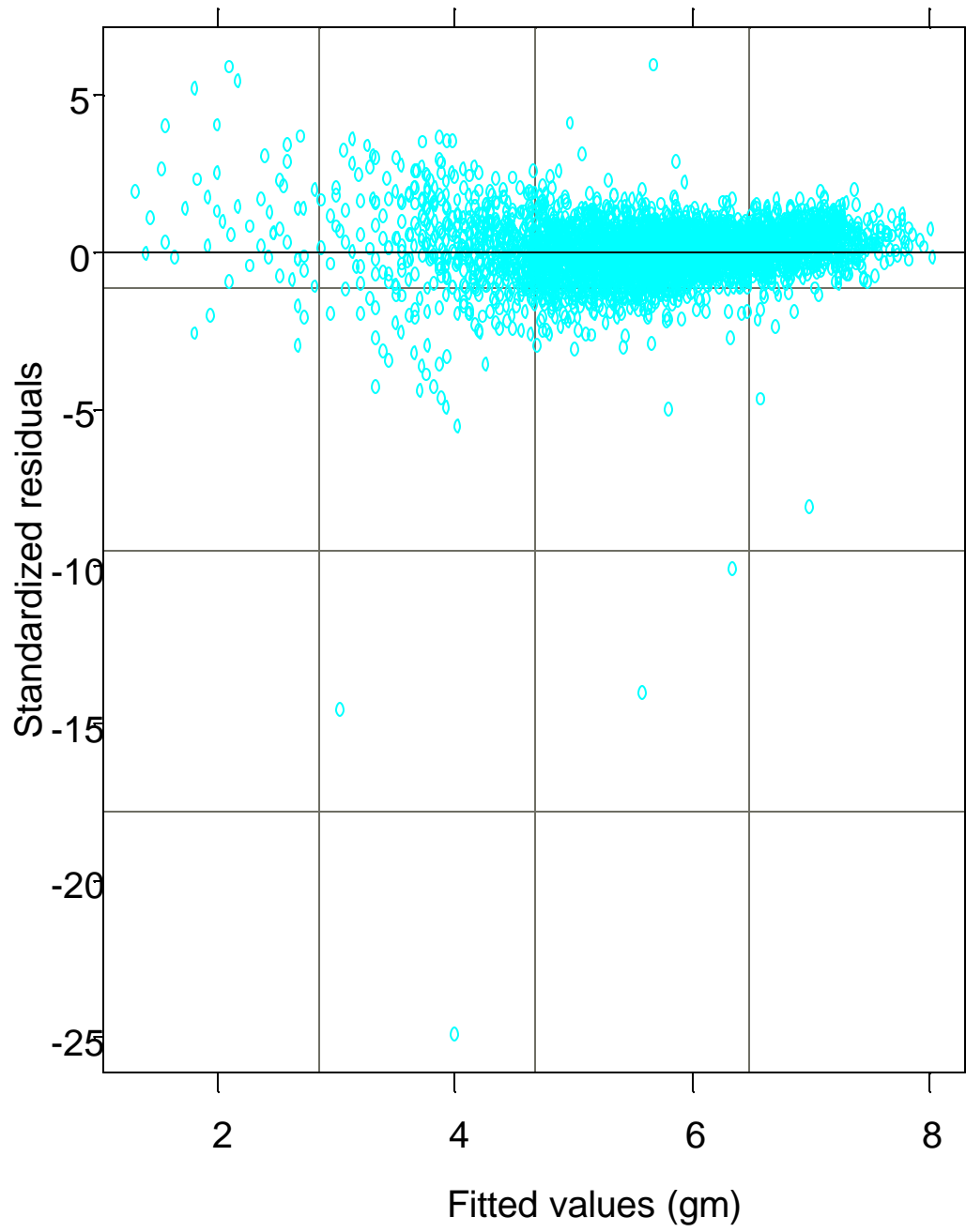
Date: 12/08/02 Time: 18:54

Sample(adjusted): 2604 4649 IF LAKE=1007

Included observations: 2046 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-11.92182	0.052653	-226.4242	0.0000
LNLENGTH	3.123478	0.009508	328.5114	0.0000
R-squared	0.981412	Mean dependent var	5.345222	
Adjusted R-squared	0.981403	S.D. dependent var	1.027029	
S.E. of regression	0.140057	Akaike info criterion	-1.092559	
Sum squared resid	40.09498	Schwarz criterion	-1.087062	
Log likelihood	1119.688	F-statistic	107919.8	
Durbin-Watson stat	1.531898	Prob(F-statistic)	0.000000	

Appendix 2



Appendix 3

Analysis of Variance Table

Response: y

Terms	Resid. Df	RSS	Test Df	Sum of Sq	F Value	Pr(F)
1 x + lake.code	5180	138.1123				
2 x	5186	146.7467	-lake.code -6	-8.634456	53.97358	0

