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① Nicely written

② Frustrations, huh?

**"Sources of Growth Analysis for the U.S. Economy"**

by

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**Economics 482**

**Winter, 1998**

What are the sources of growth in the U.S. economy in terms of real GDP? More specifically, what macroeconomic variables are the main engines in total output change? The aim of this analysis is to roughly estimate the contributions of these variables.

### The Economic Model

I use the Solow model for the theoretical relationship of the variables. The Solow model focuses on four variables: output (Y), capital (K), labor (L) and technology (A).

The production function takes the parametric form

$$(1) \quad Y(t) = F(K(t), A(t)L(t))$$

where  $t$  denotes time.<sup>1</sup> To measure the contribution of each input I transform it to

$$(2) \quad g_y = W_K g_K + W_L g_L + a$$

where  $g$  represents growth rate and  $W$  the share of income of the corresponding variables; the  $a$  term is the technological residual which can be indirectly calculated in terms of the other variables.<sup>2</sup> This second production function, is assumed albeit its monotonic transformation, to keep the constant returns to scale assumption of the first one. With this growth accounting equation we can estimate the corresponding marginal product of the factor inputs ( $W$ ); if we plug in values for their rates of growth, we can estimate the rate of growth of output; but what really creates an opportunity for a refutable hypothesis is whether there is something beyond the sum of the factors inputs' growth rates, which leads to an increase of output, given that the production function is assumed to have

<sup>1</sup> Romer, David, *Advanced Macroeconomics*, (1996), 7.

<sup>2</sup> Gillis et Al, *Economics of Development*, (1996), 46-47.

diminishing marginal returns and ultimately, convergence to a constant rate of growth.<sup>3</sup> Still, given the quarter time constraint, my aim would be limited to the estimation of the marginal products and residual, forecasting of growth rates, and if possible a breakdown of the technological residual. For now, I will focus on the estimates.

### Econometric Model

The homologous model with statistical properties for (2) is:

$$(3) \quad \log(y) = \alpha + \beta_1 \log(X_1) + \beta_2 \log(X_2) + \varepsilon$$

This model is of log-log structure which implies that a percentage change in the dependent variable will be corresponded by a percentage change in the independent variables; more specifically the  $\beta$  parameters are the constant elasticities of  $y$  with respect to  $X_{ij}$ .<sup>4</sup> The model is assumed to satisfy the necessary statistical properties so as to be a reliable technique of estimation.

### The Data

The database I use is composed of U.S. macroeconomic data - a time series going from 1952 to 2003 in a quarter time frequency.<sup>5</sup> From this database I select the most relevant variables for the model and created a new database. Although the model is composed of two independent variables, it is necessary to aggregate several variables to obtain useful series.

<sup>3</sup> Romer, David, (1996), 9-12.

<sup>4</sup> Hill et Al, Undergraduate Econometrics, (1997), 121.

<sup>5</sup> Ray, Fair, FAIRMODEL, (1997), <http://fairmodel.econ.yale.edu/>

For the labor factor a created a formula which aggregated the different labor markets, namely, federal government, state government, military and firms plus the unemployed sector. Each individual market was a multiplicative function of the number of jobs in the market, average number of worked hours, and average wages-including overtime. Total income of the unemployed sector was a product of average unemployment benefits multiplied by number of claims. *good,*

For the capital factor I also aggregated the different stock levels of capital namely, durable goods, housing, capital and inventories. These variables were a product of average prices multiplied by the number of units. The figures for both input factors' share of income were indexed to the 1992 fiscal year, and thus are in real dollars.

#### Estimation

I ran a least squares regression with the following estimation equation:

$$(4) \quad \text{LOG}(GDPR) = C(1) + C(2)*\text{LOG}(JZ) + C(3)*\text{LOG}(KZ)$$

which means that the natural log of real GDP is equal to the sum of an undetermined coefficient plus the product of a second coefficient times the natural log of the labor factor ( $JZ$ ) plus an equivalent terms for the capital factor ( $KZ$ ).

This is the output of the regression.

$$(5) \quad \text{LN}GDPR = -1.1241 + .2829 \cdot \text{LN}JZ + .6815 \cdot \text{LN}KS \quad R^2 = .995257$$
$$\quad \quad \quad (-9.0324) \quad (4.9205) \quad (12.4845) \quad (t)$$

The interpretation of these parameters is that the average shares of total income are 28% to labor and 68% to capital.

### Inference

In order to test the statistical significance of the coefficients I use a F- Test procedure which is good for assessing the reliability of multivariable models. The joint null and alternative hypothesis are:

$$(6) \quad H_0: \beta_1=0, \beta_2=0 \quad H_1: \beta_1 \neq 0, \beta_2 \neq 0$$

If the joint null hypothesis is true then  $F \sim F(J, J-K)$  where J is the number of hypothesis, and K is the number of observations. The critical value  $F_c$  comes from the  $F_{(2, 182)}$  distribution, and for the  $\alpha = .05$  level of significance it is 3.00. Running a Wald test for restricted coefficients I get an F-statistic to equal 18989.57. Since the value of the F-statistic is  $F=18989.57 > 3$ , I can reject the joint null hypothesis.<sup>6</sup> Therefore the values of the coefficients are statistically significant.

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<sup>6</sup> Hill et Al, 157-162.

## Conclusion

Within the sample and assuming no errors in the data and data structure, I can state that within a 95% confidence interval the estimates are plausible. Comparing my estimates with more reliable empirical evidence I conclude that the model needs to be revised. More reliable empirical research has concluded that the shares of total income are 70% for labor and 30% for capital.<sup>7</sup> What is left to pursue for the next draft is the correction of the model and the estimation of the growth figures and the technological residual  $\alpha$ . Furthermore, I would like to restructure the model having output as a function of percapita capital, and develop the Solow model for the steady state and its possible scenario fluctuations.

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<sup>7</sup> Mankiw, Robert, *Macroeconomics*, (1992), 74.

## References

- Romer, David, *Advanced Macroeconomics*, (1996) New York: Worth Publishers.
- Gillis et Al, *Economics of Development*, (1996) New York: W.W. Northon Publishers.
- Hill et Al, *Undergraduate Econometrics*, (1997) New York: John Wilye & Sons, Inc.
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LS // Dependent Variable is LNGDPR

Date: 02/03/98 Time: 09:36

Sample: 1952:1 1997:4

Included observations: 184

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.124123	0.124453	-9.032537	0.0000
LNJZ	0.282998	0.057513	4.920569	0.0000
LNKS	0.681540	0.054590	12.48459	0.0000
R-squared	0.995257	Mean dependent var	6.847399	
Adjusted R-squared	0.995204	S.D. dependent var	0.421295	
S.E. of regression	0.029175	Akaike info criterion	-7.052736	
Sum squared resid	0.154061	Schwarz criterion	-7.000319	
Log likelihood	390.7671	F-statistic	18989.57	
Durbin-Watson stat	0.124766	Prob(F-statistic)	0.000000	

<b>Wald Test:</b> <b>Equation: EQG</b>			
<b>Null Hypothesis:</b> C(2)=0 C(3)=0			
<b>F-statistic</b>	<b>18989.57</b>	<b>Probability</b>	<b>0.000000</b>
<b>Chi-square</b>	<b>37979.14</b>	<b>Probability</b>	<b>0.000000</b>

