“Sources of Growth:Analysis for the U.S. Economy”

by

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The aim of the research is twofold. First I estimate how the growth of real GDP is distributed between aggregate labor and aggregate capital stock in the US economy. Second I evaluate the stochastic model.

The Economic Model

I use the Solow model to establish the theoretical relationship of the variables. The model contains four variables: output (Y), capital (K), labor (L) and technology (A). Its is based on a Cobb-Dougglas production function with constant returns to scale[1]

$$(1) \qquad F(K, AL)=K^{\alpha}(AL)^{\alpha-1}$$

a multiplicative model which can be expressed in a parametric form

$$(2) \qquad Y(t) = F(K(t), A(t)L(t))$$

where t denotes time.[2] To measure the contribution of each input, it is transformed to

$$(3) \qquad \Delta Y/Y = \alpha\ \Delta K/K + (1-\alpha)\ \Delta L/L + \Delta A/A$$

where each fraction represents a percentage rate of growth of the corresponding variable[3] and the share of income is indicated by the parameter α. By definition the sum of the parameters is unitary. Equivalently we can express (3) as

$$(4) \qquad g_y = W_K g_K + W_L g_L + g_A$$

where g represents growth rate and W the share of income of the corresponding variables; the A term is the technological residual which can be indirectly calculated in terms of the other variables.[4] With this growth accounting equation we can estimate the

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[1] Romer, David, *Advanced Macroeconomics*, (1996), 7.
[2] Romer, David, *Advanced Macroeconomics*, (1996), 7.
[3] Mankiw, Robert, *Macroeconomics*, (1992), 74.
[4] Gilllis et Al, *Economics of Development*, (1996), 46-47.

corresponding marginal product (W) of the factor inputs; if we plug in values for their rates of growth, we can estimate the rate of growth of output, assuming perfect competition.[5]

Econometric Model

Making a logarithmic transformation to our the multiplicative model we can obtain the homologous stochastic model for (1) is:

$$(5) \qquad log(y) = \alpha + \beta_1 log(X_1) + \beta_2 log(X_2) + log\varepsilon$$

This model is of log-log structure which implies that a percentage change in the dependent variable due to a one percentage change in the independent variables; more specifically the β parameters are the constant elasticities of y with respect to X_i.[6] Furthermore, in accordance with the Cobb-Douglass model in terms of the factors' share of income, we can constraint the β_1 and β_2 parameters to sum to one by the following transformation:

$$(6) \qquad ln\, Y_t = \alpha + \gamma\, ln\, K_t + \beta\, ln\, L_t + ln\, \varepsilon_t$$

adding and subtracting $\gamma\, ln\, L$ to both sides of the equation and factoring out we obtain

$$(7) \qquad ln\,(Y_t/L_t) = \alpha + \gamma\, ln\,(K_t/L_t) + (\beta + \gamma - 1)\, ln\, L_t + ln\, \varepsilon_t$$

where $\beta + \gamma$ must equal one, creating the equivalent constraint of $\beta_1 + \beta_2 = 1$. The specifications of a random error term ε, along with the use of the use of the subscript t - which allows for times series values- completes our model with statistical properties.

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5 Romer, David, (1996), 9-12.
6 Hill et Al, Undergraduate Econometrics, (1997), 121.

The Data

The database I use is composed of U.S. macroeconomic data -a time series going from 1952 to 1997 in a quarter time frequency.[7] From this database I select the relevant variables for the model and create a new database. Although the model is composed of two independent variables, it is necessary to aggregate several variables to obtain useful series.

For the labor factor I create a labor force variable which is a sum of the aggregate employed and unemployed sectors over time. For the capital factor I use an aggregate of capital stock over time. Output Y, is real GDP indexed to the 1992 fiscal year. The data for all variables is given in seasonally adjusted annualized quarters, thus the corresponding time-series follows a percentage change growth pattern.

Estimation

I run an ordinary least squares regression (OLS) with the following estimation equation:

$$LOG(Y) = C(1) + C(2)*LOG(K) + (1-C(2))*LOG(L) \quad (8)$$

I then inspect for violations of the general regression model and find both heteroskedasticity and autocorrelation. To test for the presence of heteroskedasticity I use a White's test. The actual computed F-statistic critical is larger than the value F_c, therefore the null hypothesis, which implies homoskedasticity, ean be rejected. Also, I

[7] Ray, Fair, *FAIRMODEL*, (1997), http://fairmodel.econ.yale.edu/

detect strong positive autocorrelation from a correlelogram and a low Durvin-Watson statistic. Thus my regression model is not B.L.U.E. and standard errors, confidence intervals and hypothesis tests may be misleading. To correct for these violations I make another model transformation. My new model is a first order autoregressive model AR(1) model. The model has a lag structure of one period. It includes an autocorrelation coefficient ρ, which allows for decreasing correlation between coefficients; the further apart the periods are, the lower the correlation between them.[8] This way I obtain a new random error v_t which incorporates homoskedasticity into the model and corrects for strong autocorrelation. The new model is

$$(9) \qquad \ln (Y_t/L_t) - \ln(\rho Y_{t-1}/L_{t-1}) = \alpha (1-\rho) + \gamma \ln (K_t/L_t - \rho K_{t-1}/L_{t-1}) + (\beta + \gamma - 1) \ln (L_t - \rho L_{t-1}) + \ln v_t$$

The regression becomes a generalized least squares (GLS) with the following estimation equation:

(10) *LOG(Y) = C(1) + C(2)*LOG(K) + (1-C(2))*LOG(L) + [AR(1)=C(3)]*

Running the regression I obtain the following output:

(11) LOG(Y) = -.20704 + 1.5777*LOG(K) -.5777*LOG(L) + .9889 R^2=.9971

(.34) (.11) (.01) (s.e)

According to the regression, the marginal product of capital equals 157% +/- 11% of total income while labor's is nonsensical.

Inference

In order to test the statistical significance of the coefficients I use an *t*-test. The null and alternative hypothesis are:

[8] Hill et Al, 242-243.

(12) $H_0: \beta_2 = 0$ $H_1: \beta_2 \neq 0$

The critical value t_c comes from the $t_{(182)}$ distribution. The critical value for a $\alpha = .05$ level of significance it is 1.96. Since the computed value of the t-statistic is F=9.53 > 1.96 and I can reject the null hypothesis.[9] Therefore the value of the capital factor coefficient is statistically significant. Thus growth in real GDP does depend positively on the growth of the capital factor and correspondingly negatively in the growth of the labor factor.

<u>Conclusion</u>

Given the fair model population is a representative one of that of the US, and assuming no errors in the data an data structure, I can state that within a 95% confidence interval the estimates are plausible. The Solow model establishes a positive relationship between output growth and labor factor growth, thus the above results break down the economic model qualitative results. Furthermore, more reliable empirical research has concluded that the shares of total income are 70% for labor and 30% for capital.[10] Thus the model fails to predict both the institutional qualitative and quantitative results.

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[9] Hill et Al, 157-162.

[10] Mankiw, Robert, *Macroeconomics*, (1992), 74.

References

Romer, David, *Advanced Macroeconomics,* (1996) New York: Worth Publishers.

Gilllis et Al, *Economics of Development,* (1996) New York: W.W. Northon Publishers.

Hill et Al, Undergraduate Econometrics, (1997) New York: John Wilye & Sons, Inc.

Ray, Fair, *FAIRMODEL,* (1997), http://fairmodel.econ.yale.edu/

Mankiw, Robert, *Macroeconomics,* (1992) New York: Work Publishers, 74

LS // Dependent Variable is LNY
Date: 03/06/98 Time: 12:59
Sample(adjusted): 1952:2 1997:4
Included observations: 183 after adjusting endpoints
Convergence achieved after 8 iterations
LNY=C(1)+C(2)*LNK+(1-C(2))*LNL+[AR(1)=C(3)]

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-2.070390	0.034649	-59.75315	0.0000
C(2)	1.057740	0.110983	9.530623	0.0000
C(3)	0.968853	0.018661	51.91984	0.0000

R-squared	0.997193	Mean dependent var	2.288880
Adjusted R-squared	0.997162	S.D. dependent var	0.183222
S.E. of regression	0.009761	Akaike info criterion	-9.242446
Sum squared resid	0.017150	Schwarz criterion	-9.189831
Log likelihood	589.0181	F-statistic	31972.79
Durbin-Watson stat	1.336575	Prob(F-statistic)	0.000000

Inverted AR Roots	.97