Intergenerational Tax-Transfer Policies, Growth, and the Distribution of Consumption

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Abstract

In this paper we develop an overlapping-generations economy populated by mortal workers and retirees. Workers receive a stream of earnings from human capital, which consists of the stock of skills and knowledge that makes workers productive, and which grows by workers making investments in new human capital. Once acquired, human capital can be transferred to new workers entering the economy by means of an "education system". Human capital is embodied in workers, so the economy loses human capital through mortality, and through a pre-mortality event described as "retirement". The government undertakes two sorts of intergenerational transfers: i) a "younger-to-older" or "social security" transfer that insures workers against the pre-mortality loss of consumption by providing benefits to retirees, and ii) an "older-to-younger" or "education transfer" that provides "start-up" human capital to new workers entering the economy. The transfer programs are funded by taxes on earnings and consumption. With the simplifying assumptions of constant mortality and retirement hazard rates, the economy is aggregated and its growth rate derived. The growth rate is decreased by social security transfers if they are financed by taxes on the earnings, but education transfers can increase the growth rate regardless of how they are financed. The growth rate is higher if there is a greater share of consumption taxes in the tax mix, however the "optimal" tax mix is to finance social security transfers fully with consumption taxes and education transfers fully with earnings taxes. We also analyze earnings/consumption inequality in the economy, and show that the size distribution is a Pareto distribution. The impacts of the transfer programs on the mean and median values and the concentration index are derived.

Key Words: growth, human capital, overlapping generations, retirement, mortality.

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1. Introduction

We develop a simple model of an economy with overlapping generations in which growth results from the accumulation of human capital by rational but mortal agents. The human capital stock consists of the accumulated stock of skills and technological know-how embodied in the agents.[1] When new workers enter the economy, they must first be provisioned with a "start-up" level of human capital by an "education system" which transfers, or installs, some part of the existing aggregate stock of human capital (knowledge/skills/know-how) into the new workers. During their working lives, workers create and accumulate new human capital, which in turn augments the stock available for installation into future generations of new workers. Because human capital is embodied in mortal beings, it is continuously lost through death, and in our model, through a pre-mortality event denoted "retirement". Retirement consists of the random loss of human capital by workers, who then survive and consume as "retirees". Once retired, agents must depend on a social security system for consumption.[2]

Government consists of an intergenerational tax and transfer system. The "older-to-younger" transfer is an education system that installs human capital in new workers. The "younger-to-older" transfer is a social security system that provides benefits to retirees as a form of insurance against the loss of consumption resulting from the pre-mortality loss of earnings. The government finances the intergenerational transfers with taxes on the earnings of workers and on the consumption of everyone.

[1] In this paper, an expansive view of human capital is taken that encompasses knowledge capital, technology, and production know-how, including that acquired by "learning by doing" as in Arrow (1962). Lucas (1993, footnote 3) argues, rightly in our opinion, that such a view is consistent with the usage of the concept all the way back to Schultz (1962).

[2] In the simple, single-asset model developed here, retirees have no financial assets to support retirement. Obviously, it is desirable to develop a more general two-factor model, which would allow workers to accumulate financial assets as well as human capital.

We assume exponential survival functions for both the mortality and retirement hazards which make it possible to aggregate the economy and derive the aggregate growth rate. A rudimentary linear single-factor (human capital) production technology is assumed. The aggregate economy has a growing population but a stationary demography, so there is a constant dependency ratio for the social security system and a constant fraction of new workers who must be educated. We examine the effects of the tax-transfer policies and demographic parameters on the growth rate. In the special case of logarithmic utility, the growth rate is lower if higher levels of social security spending are financed with payroll taxes, but growth is higher with higher levels of education spending. The growth rate is also higher when consumption taxes constitute a larger fraction of the tax mix. We also derive the optimal tax mix, which requires fully financing social security spending with consumption taxes and fully financing education spending with earnings taxes.

Earnings and consumption are unequally distributed across agents because of different survival durations. We show that the distribution of /earnings/consumption is a Pareto distribution, and examine the impacts of demographic and policy parameters on the mean, median and concentration index of the distribution. The social security system affects only the mean and median levels of consumption in the economy, and not the level of concentration. The publicly-financed education system affects the whole distribution, with higher rates of human capital transfer corresponding to lower levels of concentration. Economies with higher education spending rates have both higher growth rates and less concentrated distributions.

Our paper fits into a literature that focuses on the role of human capital and demographic factors in the growth process. The role of human capital in the growth process was pointed out by Lucas (1988) in his critique of neoclassical models, and human capital measures were found to be significant in many empirical studies of growth, including the classic Denison (1985) study of the US experience, and the cross-country study by Barro (1991). Also, Barro and Sala-i-Martin (1992), and Mankiw, Romer and Weil (1992) found that a human-capital augmented neoclassical growth model better explained the empirical facts on cross-country growth convergence.

With the development of the so-called “new growth theory” following the seminal work of Romer (1986), researchers developed models that highlighted the role of human capital as an engine of growth through its ability to raise labor productivity. Many of these human capital growth models assume infinitely-lived or “dynastic” households which abstract from the demographic realities of birth and death among the people in which the human capital is invested. However, several empirical studies, notably Barro (1991), Barro and Lee (1994), and Kelley and Schmidt (1995) found clear negative effects of fertility and mortality rates on per capita growth rates, generating interest in “eco-demo” models that jointly model the demographic and growth forces. A classic “eco-demo” model in the economics literature is the over-lapping-generations (OLG) model. Erlich and Lui (1991), Glomm and Ravikumar (1992), Caballe (1995), Marchand, Michel and Pestieau (1996), Zhang, Zhang and Lee (2001) and Yakita (2003) all use versions of the two or three period Samuelson OLG model to analyze the roles of human capital formation, parental investment in children, fertility and mortality, and government social security and education programs on the process of economic growth.

Most find an implied negative relationship between growth and mortality, although the possibility of two-way causality is recognized. Some find a negative impact of social security and a positive impact of human capital transfers on growth.

Our model fits into a literature that uses Blanchard's (1985) "perpetual youth" OLG model, which is characterized by long-lived agents. Several authors have utilized the Blanchard OLG model to analyze economic growth and demography issues. Kalemli-Ozcan, Ryder and Weil (2000) show that an exogenous decline in mortality increases the growth rate by increasing the time that future workers spend in school, because it lengthens the time horizon over which human capital investments pay off. Boucekkine, De la Croix and Licandro (2002), and Echevarria and Iza (2006) extend the analysis with a vintage human capital and "realistic" (i.e., age-dependent) mortality hazard rates. Although mortality decline continues to induce longer schooling (and later retirement) in these models, they find an ambiguous impact on the per capital growth rate. The mortality rate-growth rate relationship is hump-shaped. Eschevarria and Iza find a negative relationship between social security spending and growth.

We maintain the original constant mortality hazard assumption, and add another constant hazard into the Blanchard model; the risk of an event we call "retirement" in which workers lose their human capital and earnings. The exogenous mortality and retirement hazard rates determine an expected life horizon, expected working horizon, and expected retirement duration for agents in the economy. Gertler (1999) and Bruce-Turnovsky (2007) use a constant retirement hazard model to analyze the effects of pay-as-you-go social security and Medicare on savings rates and welfare.

In this paper, we develop a simple and transparent model that is useful for highlighting the effects of intergenerational tax-transfer programs, such as social security and education, as well as demographic parameters, on the growth and distribution of consumption in a knowledge-based economy driven by human capital formation.

2. Demography

The economy consists of overlapping generations of mortal workers and retirees. We describe a person who has human capital that generates a stream of earnings as a "worker". A "retiree" is a person who has lost his/her human capital and is dependent on government transfers for consumption. All persons are subject to a constant mortality hazard rate *p*, and workers are subject also to a constant and independent "retirement" hazard rate π, where retirement entails the loss of human capital. Realistically, hazard rates would depend on a person's age, but age-dependent hazard rates are intractable, particularly for aggregation, so constant hazard rates (the so-called "perpetual youth" assumption) are postulated. We believe that a model based on constant hazard rates represents a useful and transparent first approximation to a more realistic aggregate economy where hazard rates are age-dependent. In this context, we interpret the hazard rates in the current model as averages for the aggregate economy.

Assuming constant and independent hazard rates, the probability of someone who entered the economy at time *s* being alive at time *z* is equal to $e^{-p(z-s)}$, and the probability of the person being alive and working (a "worker") at time *z* is $e^{-(p+\pi)(z-s)}$. We assume further that the population grows at a constant rate *n*, so at each instant $(p+n)e^{ns}$ new workers enter the economy. The fraction of the population working at any instant is

$\int_{s=-\infty}^{z} (p+n)e^{-(p+n+\pi)(z-s)}ds = \frac{p+n}{p+n+\pi}$, and the complement, $\frac{\pi}{p+n+\pi}$, is the fraction of the population retired Therefore, $\frac{\pi}{p+n}$ is the retiree "dependency" ratio for the social security program. The "fertility rate", which is the rate at which new workers enter the economy (and receive transfers of human capital by means of the education system) is $p+n$. The parameters $[p,\pi,n]$ summarize the demography of the economy.

The constant hazard rates imply expected values (time horizons) for lifetimes and working lifetimes. The expected lifetime of any agent in the economy is $1/p$ and the expected working lifetime of a worker is $1/(p+\pi)$. The duration that a worker can expect to live without productive resources (the expected length of retirement) is equal to $(1/p - 1/(p+\pi)) = (1/p)(\pi/(p+\pi)) > 0$.

3. Intergenerational Tax-Transfer Programs

The government undertakes two sorts of intergenerational transfers. Younger-to-older transfers take the form of social security consumption benefits to retirees who have lost their human capital. Older-to-younger transfers take the form of human capital transfers to new workers who enter the economy without resources and who would be unproductive without human capital.[3] The government finances the intergenerational transfers by levying taxes on earnings at rate τ^W and taxes on consumption at rate τ^C. The earnings tax falls on workers only, and distorts their human capital investment decisions, whereas the consumption tax falls on both workers and retirees is a neutral tax.

[3] Realistically, education transfers occur over an interval of time after the worker is born and before he/she enters the economy as a productive agent. We simplify by assuming the transfer is made at the instant the worker enters the economy.

Human capital pays workers a fixed after-tax rate of return $\omega\left(1-\tau^W\right)$ per unit of human capital, where ω is the gross return per unit of human capital and τ^W is the tax rate on earnings (return to human capital). The after-tax earnings of a worker with human capital h_z is $\omega\left(1-\tau^W\right)h_z$. Workers can augment their human capital by investing in new human capital and one unit of new human capital can be produced at a cost of one unit of consumption, or $\dot{h}_z = \omega\left(1-\tau^W\right)h_z - c_z^W$, where, at time *z*, h_z is the worker's human capital stock and c_z^W is the worker's consumption. This new human capital acquired by an individual also augments the aggregate stock of human capital.

Unlike some growth models, we assume that new workers are born unproductive and must be endowed with a start-up level of human capital before they enter the productive economy.[4] In particular, we assume that each worker entering the economy at time *s* receives a transfer of $\bar{h}_s$ units of human capital. The transfer is implemented through a public "education system"[5], which can install a unit of capital in a new worker at a cost of $\varepsilon \le 1$ units. The fact that ε may be less than unity reflects the possibility that human capital, which takes the form of technological skills and knowledge, can be transferred to new workers at a lower cost than it took to acquire it.[6] The assumption that $\varepsilon < 1$ allows for a sort of externality or increasing returns to human capital investment

[4] Thus, we abstract the productivity of "raw" (that is, uneducated) labor by assuming it has none.
[5] Ideally, we would also include voluntary human capital transfers from parents to new workers. Although we can easily motivate such transfers, we are unable to aggregate the economy in this case, so we must, unfortunately, ignore voluntary transfers in this model.
[6] To simplify, we assume that the human capital transfers occur to new workers only, while existing workers augment human capital only through production of new human capital. More generally, existing workers add human capital both through production and transfers (i.e., by inventing new skills/knowledge and copying the skills and knowledge of others). To properly model this more general human capital production and transfer system, some form of vintage human capital model would need to be specified.

that other authors have introduced in other ways, for example through the production technology (Romer, 1986).

Workers randomly retire (lose their human capital). After retirement, persons are provided with a social security benefit based on their past earnings, and which possibly grows after retirement. The initial social security benefit is equal to a fraction β of the worker's earnings at the time of retirement. We describe β as the "social security replacement rate".[7]

Together, the older-to-younger and younger-to-older transfers capture what we believe to be the important features of any knowledge-based economy. Human capital, in the form of production know-how, is embodied in workers, and is risky and depreciating for any individual because human capital is subject to loss through retirement and mortality. Human capital is lost to the economy through retirement and death, but it can be transferred, at a cost, to new workers. In this way, human capital is "immortal" and continuously productive although the agents in which it is embodied are not. However, there are overall "economies" in the human capital creation and transfer process that arise from the fact that human capital, once created, can be copied or replicated by others at a lower cost. We postulate that this ability to transfer human capital (to "stand on the shoulders of giants") is an important determinant underlying the growth of economies.

4. The Human Capital Investment Decision

Growth in the economy is determined by the accumulation of new human capital by workers, as well as the productivity of human capital, demographic factors, and the

[7] In the US social security system, the replacement rate is determined by a progressive formula applying to a worker's indexed earnings averaged over a work history of up to 35 years. The simplified assumption of the model captures the fact that a worker's benefit at the time of retirement is increased by the level of earnings.

intergenerational transfer programs. To determine the human capital investment decisions by workers, we first consider how a worker's expected retirement utility, $U^R(h_t)$, depends on the level of his/her human capital stock at the time of retirement given that the social security benefit is based on previous earnings. In particular, for a household that retires at time *t*:

$$U^R(h_t) = \int_{z=t}^{z=\infty} e^{-(p+\delta)(z-t)} u\left(c_z^R(h_t)\right) dz \tag{1}$$

and $c_z^R(h_t) = \dfrac{\beta \omega h_t e^{g^B(z-t)}}{1+\tau^C}$.

The time discount rate is δ, $c_z^R(h_t)$ is the retiree's consumption at time $z \geq t$, h_t is the worker's human capital at the time of retirement *t*, and τ^C is the consumption tax rate. After retirement, persons have no earnings and consume the transfer from the government which is a fraction β of their before-tax earnings at the time of retirement (ωh_t). After retirement, the social security benefit grows at rate g^B. Later, in order to analyze a steady state, we let g^B equal the per capita growth rate of the economy.

With iso-elastic utility $u\left(c_z^R\right) = \dfrac{1}{\eta}\left(c_z^R\right)^\eta$, where $\dfrac{1}{1-\eta}$ is the inter-temporal elasticity of substitution (IES) and $\eta < 1$, we can integrate equation (1) to obtain:

$$U^R(h_t) = \frac{1}{\eta}\left(\frac{\beta \omega h_t}{1+\tau^C}\right)^\eta \frac{1}{p+\delta-\eta g^B} \tag{1'}$$

for $p+\delta > \eta g^B$. (Since, typically, $\eta \leq 0$, this latter condition is satisfied.)[8]

[8] In the log utility case ($\eta = 0$), equation (1') is $U^R(h_t) = \dfrac{\ln\left[\beta \omega h_t / \left(1+\tau^C\right)\right]}{p+\delta} + \dfrac{g^B}{\left(p+\delta\right)^2}$

and $\dfrac{\partial U^R(h_t)}{\partial h_t} = \dfrac{\eta U^R(h_t)}{h_t}$.

We now consider a worker's human capital investment decision. A worker entering the economy at time *s* invests in human capital so as to:

$$\max_{c_t^W} U^W(\bar{h}_s) = \int_{t=s}^{\infty} e^{-[p+\delta+\pi](t-s)} \left\{ u(c_t^W) + \pi \cdot U^R(h_t) \right\} dt \tag{2}$$

subject to

$\dot{h}_t = \omega(1-\tau^W) h_t - c_t^W (1+\tau^C)$ and $h_t = \bar{h}_s$ at $t = s$.

The "felicity" of the worker at time *t*, is equal the time and hazard discounted utility from working consumption $e^{-[p+\delta+\pi]} u(c_t^W)$ plus the time and mortality hazard discounted lifetime utility in retirement, $e^{-[p+\delta](t-s)} U^R(h_t)$, times the probability of becoming retired at *t*, $\pi e^{-\pi(t-s)}$.

The first order conditions for problem (2) can be summarized as:

$$\frac{-u''(c_t^W)\cdot \dot{c}_t^W}{u'(c_t^W)} = \omega(1-\tau^W) - (p+\delta+\pi) + \pi \left[\frac{(1+\tau^C)\partial U^R(h_t)/\partial h_t}{u'(c_t^W)} \right]. \tag{3}$$

With the simplifying assumption of an iso-elastic utility function, equation (3) can be solved for the steady state of an individual worker in which c_t^W and h_t grow at a time constant rate $\sigma = \dfrac{\dot{c}_t^W}{c_t^W} = \dfrac{\dot{h}_t}{h_t}$. Let $\gamma = \dfrac{c_t^W (1+\tau^C)}{h_t}$ denote the time constant "expenditure rate" (that is, consumption inclusive of tax as a fraction of the agent's human capital stock), then σ and γ satisfy[9]

[9] In the log utility case ($\eta = 0$), $\gamma = p + \delta$, so the expenditure rate is independent of all policy parameters, and $\sigma = \omega(1-\tau^W) - (p+\delta)$, so the accumulation rate depends on the earnings tax rate only.

$$\sigma = \omega\left(1-\tau^{W}\right)-\gamma \tag{4.1}$$

$$\sigma = \frac{1}{1-\eta}\left[\omega\left(1-\tau^{W}\right)-\left(p+\delta+\pi\right)+\kappa\cdot\gamma^{1-\eta}\right]. \tag{4.2}$$

where $\kappa = \dfrac{\pi\omega^{\eta}\beta^{\eta}}{p+\delta-\eta g^{B}}$. Equations (4.1) and (4.2) determine a unique solution for σ and γ per Figure 1.

For a given value of g^{B}, we now establish Propositions 1 and 2.

Proposition 1:

Workers accumulate human capital at a rate which is i) independent of the consumption tax rate, ii) decreasing in the earnings tax rate, and iii) decreasing (increasing) in the social security replacement rate when the IES is less (greater) than unity. In the case of logarithmic utility ($\eta = 0$), the accumulation rate is independent of the social security replacement rate.

Proposition 2:

The expenditure rate is i) independent of the consumption tax rate, ii) decreasing (increasing) in the earnings tax rate when the IES is less (greater) than unity, and iii) increasing (decreasing) in the social security replacement rate when the IES is less (greater) than unity. In the case of logarithmic utility ($\eta = 0$), the expenditure rate is independent of all policy parameters.

The independence of the accumulation and expenditure rates from the consumption tax rate follows directly from equations (4.1) and (4.2). The remaining relationships are found by totally differentiating (4.1) and (4.2) to obtain:

$$\frac{\partial\sigma}{\partial\tau^{W}} = \frac{-\omega}{1-\eta}\left[\frac{1+\left(1-\eta\right)\cdot\kappa\cdot\gamma^{-\eta}}{1+\kappa\cdot\gamma^{-\eta}}\right] < 0 \tag{5.1}$$

$$\frac{\partial\gamma}{\partial\tau^{W}} = \frac{\omega\cdot\eta}{\left(1-\eta\right)\left(1+\kappa\cdot\gamma^{-\eta}\right)} >=< 0 \text{ as } \eta >=< 0 \tag{5.2}$$

$$\frac{\partial \sigma}{\partial \beta} = -\frac{\partial \gamma}{\partial \beta} = \frac{\eta \kappa \gamma^{(1-\eta)}}{(1-\eta)\beta\left(1+\kappa\gamma^{-\eta}\right)} >=< 0 \text{ as } \eta >=< 0 \tag{5.3}$$

Although all workers have the same accumulation and expenditure rates, they have different human capital stocks and consumption levels because they are born at different times and have accumulated human capital over different intervals. In a later section, we examine the resulting distribution of consumption.

5. The Aggregate Economy

The assumption of constant hazard rates affords straightforward, albeit tedious, aggregation of variables.

5.1 Aggregate Production

Given the human capital accumulation rate σ, a worker who entered the economy at time *s* with initial human capital $\bar{h}_s$ has human capital $h_z(s) = \bar{h}_s e^{\sigma(z-s)}$ at time $z \geq s$. At time *z*, the number of workers who entered the economy at *s* and are still alive and working is $N_z^W(s) = (p+n)e^{nz}e^{-(p+\pi+n)(z-s)}$. The aggregate human capital stock at time *z* is:

$$H_z = (p+n)e^{nz} \int_{s=-\infty}^{z} \bar{h}_s e^{[\sigma-(p+\pi+n)](z-s)} ds . \tag{6}$$

Assuming a simple linear or "AK" production function, aggregate output is

$$Q_z = \omega H_z \tag{7}$$

Differentiating (6), the growth rate of the aggregate human capital stock and output is:

$$g = \frac{\dot{H}_z}{H_z} = \frac{\dot{Q}_z}{Q_z} = \left[\sigma - (p+\pi)\right] + \frac{(p+n)\bar{h}_z e^{nz}}{H_z} . \tag{8}$$

That is, the growth rate of the aggregate production depends on the rate of human capital accumulation by workers, the rate at which human capital is lost through death and retirement, and the rate at which human capital is transferred to new workers. We now assume that workers entering the economy receive collectively, through the education transfer, a start-up human capital level equal to some fraction θ of the existing aggregate human capital stock[10], so $\bar{h}_z = \frac{\theta}{(p+n)e^{nz}} H_z$ for all z. This assumption implies that $\dot{\bar{h}}_z / \bar{h}_z = g - n$, so the start-up human capital of new workers grows at the growth rate of the per capita stock.

In this case, the growth rate of the human capital stock and aggregate production in the economy is:

$$\begin{aligned} g &= \sigma - (p+\pi) + \theta \\ &= \omega\left(1-\tau^W\right) - \gamma - (p+\pi) + \theta \end{aligned} \quad (9)$$

From (9), we see that the growth rate of the aggregate economy depends on the after-tax return on human capital $\omega\left(1-\tau^W\right)$, the worker saving ($\sigma$) or expenditure rate ($\gamma$) rates, the demographic parameters ($p+\pi$), and the human capital transfer rate (θ).

5.2 Aggregate Consumption

Since the expenditure rate is the same for all workers, it follows that aggregate consumption expenditure by workers is equal to

$$C_z^W\left(1+\tau^C\right) = \gamma H_z \quad (10)$$

[10] This reflects what we hope is a realistic assumption about the education sector, which is that at any instant of time the education system transfers to the entering new workers a fraction of the current stock (i.e., augmented by past accumulations) of human capital (production/technology skill sets).

Aggregate consumption expenditure by retirees is equal to aggregate social security program benefits. To determine aggregate benefits, note that the social security benefit at time z to a worker who retired at time $t \le z$ and who entered the economy at $s \le t$, is $b_z(s,t) = \beta\omega\bar{h}_s e^{\sigma(t-s)} e^{g^B(z-t)}$. The number of such retirees is $N_z^R(s,t) = (p+n)e^{ns}\pi e^{-(p+\pi)(t-s)} e^{-p(z-t)}$, so aggregate social security program spending (retiree consumption expenditure) at time z is

$$\begin{aligned} C_z^R\left(1+\tau^C\right) = B_z &= \int_{t=-\infty}^{z} dt \int_{s=-\infty}^{t} N_z^R(s,t)\cdot b_z(s,t)\,ds \\ &= \frac{\beta\omega\pi}{g+p-g^B} H_z \end{aligned} \tag{11}$$

To simplify, we consider the special case in which an individual's post-retirement social security benefit grows at the per capital growth rate, that is $g^B = g - n$. In this case, equation (11) reduces to:

$$C_z^R\left(1+\tau^C\right) = B_z = \frac{\pi\beta\omega}{p+n} H_z . \tag{11'}$$

The term $\frac{\pi\beta\omega}{p+n}$ is the *gross* social security benefit rate (that is, disregarding revenue recovered from taxes on retiree consumption spending). From (11') we see that, for a given level of H_z, the level of social security benefits depends on the product of the social security replacement rate β and the program dependency ratio $\frac{\pi}{p+n}$.

Finally, the ratio of aggregate retiree to aggregate worker expenditure is:

$$\frac{C_z^R\left(1+\tau^C\right)}{C_z^W\left(1+\tau^C\right)} = \frac{\pi\beta\omega/\gamma}{p+n} \tag{12}$$

and total consumption expenditure in the economy equals

$$C_z\left(1+\tau^C\right)=C_z^W\left(1+\tau^C\right)+C_z^R\left(1+\tau^C\right)$$
$$=\left(\gamma+\frac{\pi\beta\omega}{p+n}\right)\equiv\hat{\gamma}H_z \qquad (13)$$

where $\hat{\gamma}=C_z\left(1+\tau^C\right)/H_z$ denotes the aggregate expenditure rate as a fraction of the human capital stock.

Proposition 3:

All aggregate variables in the economy grow at the growth rate of the aggregate human capital stock as given by equation (9).

6. The Determinants of Growth

To describe the impacts of policy parameters on growth, it is first necessary first to determine an expression for the government budget constraint. Total tax collections at any instant, $T_z=\tau^W\omega H_z+\tau^C C_z$, must be sufficient to fund government spending on social security benefits $B_z=\left[\pi/\left(p+n\right)\right]\beta\omega H_z$ and the cost of human capital transfers $E_z=\varepsilon\theta H_z$.[11] Dividing by H_z, and rearranging we can express the government budget constraint in terms of program and tax rates as

$$\tau^W\omega+\hat{\tau}^C\gamma=\frac{\pi\beta\omega}{p+n}\left(1-\hat{\tau}^C\right)+\varepsilon\theta \qquad (15)$$

where $\hat{\tau}^C=\tau^C\big/\left(1+\tau^C\right)$ is the tax-inclusive consumption tax rate, τ^W is the earnings tax rate, β is the social security replacement rate, and θ is the human capital transfer (education) rate.[12] The government budget constraint implies that only three of the

[11] Since there are no financial assets, the government must balance its budget at each instant of time.
[12] The government budget constraint can also be derived from the aggregate output constraint. This constraint states that *gross* investment in human capital (net investment plus replacement of that lost through death and retirement net of recapture through the education system) is equal to output minus consumption minus the cost of education, or

policy parameters can be chosen independently, with the fourth parameter set to balance the budget. The government budget constraint, the growth equation

$$g = \omega\left(1-\tau^{W}\right) - \gamma - \left(p+\pi\right) + \theta \tag{9}$$

and the equation determining the workers' accumulation rate

$$\gamma = \frac{1}{1-\eta}\left\{\left(p+\delta+\pi\right) - \left(\frac{\pi\omega^{\eta}\beta^{\eta}}{p+\delta+\eta n-\eta g}\right)\cdot\left[\gamma\right]^{1-\eta}\right\} - \frac{\eta\omega\left(1-\tau^{W}\right)}{1-\eta} \tag{4}$$

can be solved to determine the impacts of all model and policy parameters on the growth rate. Equation (4) is obtained by combining equations (4.1) and (4.2) and setting $g^{B} = g - n$.[13] Unfortunately, the impacts of the policy parameters on the growth rate are not apparent from this system of non-linear equations. However, if we assume $\eta = 0$ (the log utility case), $\gamma = p + \delta$ and we can establish the following proposition:

Proposition 4:

i) An increase in the social security replacement rate β decreases the growth rate if financed by an increase in the tax on earnings, and has no impact on the growth rate if financed by a tax on consumption.

ii) An increase in the human capital transfer (education) rate θ increases the growth rate if financed by an increase in the tax on consumption. It also increases the growth rate, but by a lesser amount, if financed by an increase in the tax on earnings and the cost of transferring human capital ε is less than the cost of acquiring new human capital (unity).

iii) A revenue neutral change in the tax structure that decreases the tax on the return to human capital increases the growth rate of the economy.

$\dot{H}_z + \left(p+\pi-\theta\right)H_z = Q_z - C_z - \varepsilon\theta H_z = \left(\omega - \left[\gamma + \pi\beta\omega\big/\left(p+n\right)\left(1+\tau^{C}\right)\right] - \varepsilon\theta\right)H_z$. Setting $\dot{H}_z = gH_z$ and substituting (9) yields (15).

[13] Equation (4) can also be expressed in terms of the accumulation rate by substituting $\gamma = \omega\left(1-\tau^{W}\right) - \sigma$

iv) The growth maximizing tax structure is one where government spending is financed to the maximum extent possible by consumption taxes. This implies a growth maximizing tax structure that is fully reliant on consumption taxes

Neither the social security replacement rate nor the consumption tax rate directly enters the growth rate equation (9), so an increase in β financed with an increase in τ^C consistent with (15) leaves the growth rate unchanged. However, if the increase in the replacement rate is financed by an increase in τ^W, the growth rate is decreased according to (9). The human capital transfer rate θ affects the growth rate directly in equation (9), so an increase financed with an increase in τ^C must increase the growth rate. If the increase in θ is financed by an increase in τ^W, equation (15) implies that $\partial\left(\omega\tau^W\right)/\partial\theta = \varepsilon$, so the growth rate is increased by $1-\varepsilon$, which is positive if $\varepsilon < 1$. Further, an increase in τ^W decreases the growth rate from (9), and if balanced by an increase in τ^C to satisfy (15), the net effect on growth is negative because the increase in the consumption tax rate increase does not affect the growth rate. It follows then that the growth rate is maximized by raising τ^C to its maximum level and lowering τ^W to its minimum level. Providing $p+\delta > \varepsilon\theta$, this implies $\tau^W = 0$. To see this, set $\tau^W = 0$ and rearrange (15) as

$$\hat{\tau}^C = \frac{\pi\beta\omega/\left(p+n\right)+\varepsilon\theta}{\pi\beta\omega/\left(p+n\right)+p+\delta}. \qquad (15')$$

Since the tax inclusive consumption tax rate $\hat{\tau}^C$ cannot exceed one, τ^W is zero only if the cost of the education spending rate $\varepsilon\theta$ is less than the worker expenditure rate $p+\delta$.

From (9) and (15), it is apparent also that the demographic parameters directly and indirectly affect the growth rate. Higher mortality, fertility and retirement rates all directly reduce the growth rate. However, these demographic parameters also affect the government budget constraint, mainly through social security dependency ratio, but also through the worker consumption rate. Combined with the government budget, the levels of the demographic parameters can have ambiguous effects on growth overall. For example, higher values of p and n reduce the dependency ratio and the earnings tax rate needed to support a given social security replacement rate. To the extent that the lower dependency ratio results in a lower earnings tax rate (for instance, as in the US where social security benefits are funded with a dedicated payroll tax), the effect of the higher mortality and fertility rates have ambiguous effects on the growth rate. This is especially the case when the social security replacement rate is high.[14]

7. The Size Distribution of Consumption

Although workers have identical accumulation and expenditure rates, the level of earnings and consumption varies across workers according to their age.[15] Similarly, retiree consumption levels vary according to the level of their social security benefits, which depend on their earnings at the time of retirement, which depends on the lengths of their work histories.

[14] A similar result was found by Zhang, Zhang and Lee (2001) using a Samuelson type OLG model.
[15] Because there are no financial assets in the model, the distribution of earnings and consumption by workers is same. With financial assets, workers could choose a smoother consumption path than the earnings path.

7.1 The Size Distribution of Worker Consumption

In Appendix, we show that the size distribution of workers' consumption is a Pareto distribution of the form:[16]

$$F^W\left(\hat{c}_z\right) = 1 - \left(\frac{\bar{c}_z}{\hat{c}_z}\right)^{\frac{\theta}{p+\pi+n-\theta}} \tag{16.W}$$

where $0 \leq F^W\left(\hat{c}_z\right) \leq 1$ is the share of total worker consumption undertaken by workers consuming at a rate less than $\hat{c}_z$. The minimum (and modal) level of worker consumption in the economy at time *z* is $\bar{c}_z^W = \gamma \bar{h}_z$, which is that of a new worker. Note that $F^W\left(\bar{c}_z\right) = 0$. Further, in order for the size distribution to have finite mean[17], it is necessary that $2\theta > p + \pi + n > \theta$. Since average worker consumption is $\mu\left(c_z^W\right) = C_z^W\left(\frac{p+n+\pi}{p+n}\right)$, this condition is equivalent to the requirement that $\mu\left(c_z^W\right) > \bar{c}_z^W > \frac{1}{2}\mu\left(c_z^W\right)$. That is, the education transfer must provide new workers with a level of start-up human capital sufficient to consume at a level that is less than the mean worker consumption but more than half of mean worker consumption.

The mean, median and Gini ratio for this Pareto distribution are, respectively[18]:

[16] Wold and Whittle (1957) first showed that differences in the length of time over which agents accumulate assets can lead to a Pareto distribution of wealth. More recently, Reed (2001) demonstrates that exponentially distributed durations can explain power-law tail behavior (such as that of the Pareto distribution) in models where variables, such as wealth, evolve according to Gibrat's law of proportional effects, such as in this model.
[17] It is well-known that the Pareto distribution can have a heavy tail and exhibit extreme values.
[18] See Aaberge (1999) and Weinstein (2005).

$$\mu\left(c_z^W\right)=\left[\frac{\theta}{2\theta-\left(p+\pi+n\right)}\right]\overline{c}_z^W>0 \text{ if } 2\theta>p+\pi+n$$

$$\text{med}\left[c_z^W\right]=2^{\frac{p+\pi+n-\theta}{\theta}}\cdot\overline{c}_z^W \qquad (17.W)$$

$$G^W=\frac{p+\pi+n-\theta}{3\theta-\left(p+\pi+n\right)}<1 \text{ as } 2\theta>p+\pi+n\,.$$

From (17.W), we see that the distribution of worker wealth depends on $\left[p,\pi,\theta,n,\overline{c}_z^W\right]$. For a given value of $\overline{c}_z^W$, a higher human capital transfer rate θ decreases both mean and median worker consumption, but also decreases the Gini measure of concentration.[19] Concentration is decreased because the education transfer program taxes unequal earnings and consumption levels, and distributes the proceeds equally to new workers in the form of start-up human capital. The evening out of initial opportunities is, of course, a traditional justification of public education systems. An increase in any of the demographic parameters (π,p,n) increases mean and median consumption level, and unambiguously increases the concentration index.

The distribution of worker consumption is not affected directly by social security transfers, but an increase in the consumption tax rate decreases the mean and median worker consumption by decreasing $\overline{c}_z^W$.

7.2 The Size Distribution of Retiree and Overall Consumption

The distribution of retiree consumption is determined by the distribution of social security benefits. In the Appendix, we show that the distribution of retiree benefits is

[19] The increase in θ would also unambiguously increase c_z^W if financed by an increase in the tax on earnings, so the effect on mean and median worker consumption is ambiguous overall.

also a Pareto distribution. Since $c_z^R = \dfrac{b_Z}{1+\tau^C}$ for a retiree receiving social security benefit b_z, the distribution of retiree consumption is given by:

$$F^R\left(\hat{c}_z^R\right) = 1 - \left(\frac{\overline{c}_z^R}{\hat{c}_z^R}\right)^{\frac{\theta}{p+\pi+n-\theta}} \qquad (16.R)$$

where $\overline{c}_z^R = \dfrac{\overline{b}_z}{1+\tau^C}$ and $\overline{b}_z = \beta\omega\overline{h}_z$ is the minimum (modal) social security benefit. The mean. median and Gini concentration coefficient for the retiree consumption distribution are, respectively:

$$\mu\left(c_z^R\right) = \left[\frac{\theta}{2\theta - \left(p+\pi+n\right)}\right]\overline{c}_z^R > 0 \text{ if } 2\theta > p+\pi+n$$

$$\text{med}\left[c_z^R\right] = 2^{\frac{p+\pi+n-\theta}{\theta}} \cdot \overline{c}_z^R \qquad (17.R)$$

$$G^W = \frac{p+\pi+n-\theta}{3\theta - \left(p+\pi+n\right)} < 1 \text{ as } 2\theta > p+\pi+n\,.$$

The distribution of retiree consumption is affected in the same way as the worker consumption distribution by the education transfer program and demographic parameters, but $\overline{c}_z^R$ is directly increased by an increase in the social security replacement rate β as well as by a decrease in the consumption tax rate. The concentration of the retiree consumption distribution is not directly affected by the social security or tax parameters.

The distributions of worker and retiree consumptions can be combined, so that the overall distribution of consumption is the economy is given by the Pareto distribution:

$$F\left(\hat{c}_z\right) = 1 - \left(\frac{\overline{c}_z}{\hat{c}_z}\right)^{\frac{\theta}{p+\pi+n-\theta}} \qquad (16)$$

where $\bar{c}_z = \left\{ \left[\bar{c}_z^W \left(1-\gamma^R\right)^{\frac{1}{\vartheta}} \right]^{\vartheta} + \left[\bar{c}_z^R \left(\gamma^R\right)^{\frac{1}{\vartheta}} \right]^{\vartheta} \right\}^{\frac{1}{\vartheta}}$, $\vartheta = \dfrac{\theta}{p+\pi+n-\theta}$, and

$\gamma^R = \dfrac{\pi\beta\omega}{\left(p+n\right)\gamma + \pi\beta\omega}$ is the ratio of aggregate retiree consumption to aggregate consumption. Thus, the only policy parameter that affects the concentration of the consumption distribution is the education transfer rate, but the mean and median consumption levels in the economy are affected, in a complicated way, by the social security replacement rate, the consumption tax rate, and the education transfer rate.

We summarize the results of this section in the following proposition:

Proposition 5:

The size distribution of consumption is unequal in the economy and described by a Pareto distribution. The concentration of the distribution is not directly affected by the social security system or the tax structure, but is decreased by an increase in the human capital transfer or by a decrease in any of the demographic parameters. Mean and median consumption by workers and retirees is increased by a decrease in the human capital transfer rate or by an increase in the demographic parameters. Since higher rates of human capital transfer and lower mortality, retirement and fertility rates increase the growth rate, these relationships imply a that a negative relationship between growth and consumption inequality.

8. The Optimal Tax Mix

Earlier we saw that the growth maximizing tax structure was one that maximizes reliance on consumption taxes. However, growth maximization does not imply welfare maximization. In this section we derive the welfare maximizing mix of consumption and earnings taxes in the economy and show that it is a particular mix of consumption and earnings taxes. Throughout this section, we maintain the assumption of logarithmic utility ($\eta = 0$).

We can substitute the optimal consumption and savings plan of a worker into his/her objective function equation (2) to derive the value of maximized lifetime expected utility. Let $x_z = c_z\left(1+\tau^C\right)$ denote consumption expenditure gross of consumption tax. For a worker making consumption expenditure $\hat{x}_z$ at time z, after integration the resulting value function with logarithmic utility is:

$$V_z^W\left(\hat{x}_z\right) = \frac{\ln\left[\dfrac{\hat{x}_z}{1+\tau^C}\right] + \dfrac{\pi}{p+\delta}\ln\left[\dfrac{\beta\omega}{p+\delta}\cdot\dfrac{\hat{x}_z}{1+\tau^C}\right] + \dfrac{p+\delta+\pi}{\left(p+\delta\right)^2}\cdot\omega\left(1-\tau^W\right)}{p+\delta+\pi} + \Gamma^W \qquad (18)$$

where Γ^W is a constant not involving tax rates. From equation (1), we can express the lifetime expected utility of a retiree making consumption expenditures $\hat{x}_z$ at time *z* as:

$$V_z^R\left(\hat{x}_z\right) = \frac{\ln\left[\dfrac{\hat{x}_z}{1+\tau^C}\right] + \dfrac{\omega\left(1-\tau^W\right)}{p+\delta}}{p+\delta} + \Gamma^R \qquad (19)$$

where Γ^R is a constant not involving tax rates. We now state the following proposition regarding the optimal tax mix for the economy:

Proposition 6:

The optimal tax mix in the economy is to use consumption taxes to finance the social security transfers to retirees and the earnings tax to finance human capital transfers (education) to new workers. The described tax mix is shown to be optimal in terms of a utilitarian social welfare function, but the result obtains for any increasing, additively separable social welfare function.

To prove the proposition, we examine the problem of maximizing a utilitarian social welfare function[20]

$$\Psi_z = \int_{\hat{x}_z=\bar{x}_z}^{\infty} N_z^W(\hat{x}_z)\cdot V_z^R(\hat{x}_z)\,d\hat{x}_z + \int_{\hat{x}_z=\bar{x}_z}^{\infty} N_z^R(\hat{x}_z)\cdot V_z^R(\hat{x}_z)\,d\hat{x}_z \qquad (20)$$

where $N_z^W(\hat{x}_z)$ is the number of workers at time *z* making consumption expenditures $\hat{x}_z$, $N_z^R(\hat{x}_z)$ is the number of retirees making consumption expenditures $\hat{x}_z$ at time *z*, $\bar{x}_z^W = \bar{c}_z^W(1+\tau^C)$ is the minimum consumption expenditure of a worker at time *z*, and $\bar{x}_z^R = \bar{c}_z^R(1+\tau^C)$. We maximize (20) over $[\tau^C, \tau^W]$ subject to the aggregate period government budget constraint[21]

$$\frac{\tau^C}{1+\tau^C}\left[X_z^W + \bar{B}_z\right] + \tau^W \omega H_z - \bar{B}_z - \bar{E}_z = 0 \qquad (21)$$

where X_z^W is aggregate consumption expenditure by workers at time *z* and $\bar{B}_z = X_z^R$ is aggregate consumption expenditure by retirees (equal to the aggregate social security program spending, which is assumed fixed), and $\bar{E}_z$ is education program spending, also assumed fixed.

The first order conditions with respect to τ^C and τ^W can be expressed:

---

[20] Note in what follows that the actual values of $N_z^W(\hat{x}_z)$ and $N_z^R(\hat{x}_z)$ do not matter. Any positive continuous set of weights on the individual utilities will give the same result. The optimal tax mix result depends on the additively separable property of the social welfare function but not on the specific utilitarian form.

[21] Note that once the optimal values τ^C and τ^W are chosen, they satisfy all future period government budget constraints because all aggregate variables including program spending are growing at the same rate.

$$\frac{N_z^W + N_z^R}{p+\delta} = \frac{\phi}{1+\tau^C}\left[X_z^W + B_z\right] \tag{22.1}$$

$$\frac{N_z^W + N_z^R}{\left(p+\delta\right)^2} = \phi H_z \tag{22.2}$$

where $\phi > 0$ is the multiplier on the government budget constraint,

$N_z^W = \int_{\hat{x}_z = \bar{x}_z^W}^{\infty} N_z^W\left(\hat{x}_z\right) d\hat{x}_z = e^{nz}\frac{p+n}{p+\pi+n}$ is the number of workers at time z, and

$N_z^R = \int_{\hat{x}_z = \bar{x}_z^R}^{\infty} N_z^R\left(\hat{x}_z\right) d\hat{x}_z = e^{nz}\frac{\pi}{p+\pi+n}$ is the number of retirees at time z. Using equations (11) and (21), first order conditions (22.1) and (22.2) reduce to

$$\tau^C\left(C_z^W + \frac{\bar{B}_z}{1+\tau^C}\right) = \bar{B}_z \text{ and } \tau^W \omega H_z = \bar{E}_z \tag{22.3}$$

In words, in order to maximize social welfare in the economy, social security program spending should be entirely financed by consumption taxes and education program spending should be entirely financed by earnings taxes.

9. Conclusions

Our purpose in this paper is to develop a simple model of overlapping generations that incorporates some basic demographic facts of birth and death, the creation, loss, and transfer of capital embodied in humans, the subsequent growth rate of the aggregate economy, and the inequality of consumption across individuals. The simplifying assumptions of constant and independent hazard rates, while at odds with facts in the case of any one individual, allows us to describe and analyze the properties of the aggregate economy. Overall, we subscribe to the view, effectively expressed by Blanchard (1985), that the payoffs in terms of the ability to aggregate variables and the insights offered into the workings of the aggregate economy more than compensate for the obvious unrealism of the exponential survival functions. Less justifiable are the assumptions of a single type of capital and the restriction of a logarithmic utility function to derive certain results. It is our hope that further research can generalize the model in these regards.

The resulting economy is one in where growth depends on the formation of new human capital embodied in new human beings in the face of human capital loss through retirement and death. The model predicts that substantial inequalities in consumption can arise simply from differing survival rates, allowing some agents to work and accumulate human capital for longer durations than others. Interestingly, social security programs that insure against this human capital risk do not affect the concentration of the consumption distribution, whereas education programs reduce the concentration.

The model provides support for the commonly-held views that social security transfer programs, as necessary as they are to insure consumption against loss of worker productivity in old age, are detrimental to growth, particularly when financed with payroll taxes, while education programs that prepare new workers for the economy will enhance growth and reduce inequalities. Finally, the model implies, counter to practice in most countries, that social security transfers should be financed by consumption taxes, not payroll taxes, and that taxes on earnings are desirable only to the extent that those taxes are used to enhance the human capital stock through the transfer of knowledge and skills to new workers.

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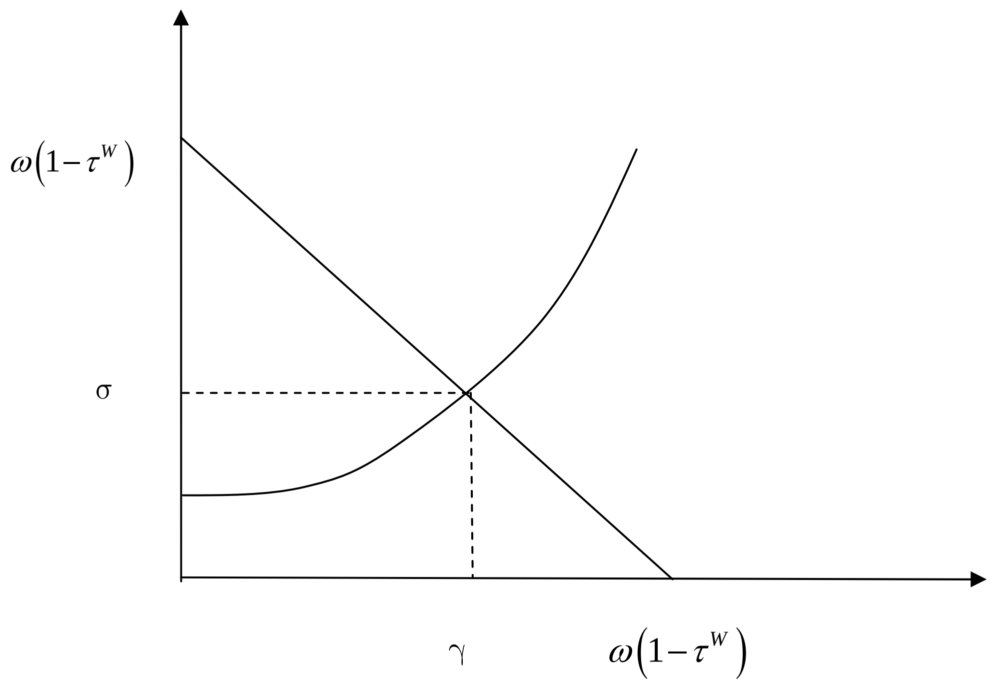
Figure 1



Appendix: Derivation of the Size Distribution of Consumption

In this paper it is claimed that the size distribution of consumption that results in this economy is a Pareto distribution. In this Appendix, we provide the algebraic details.

A.1. The Size Distribution of Workers' Consumption

The consumption level of a worker depends on the time he/she entered the economy. A worker at time *z* who entered the economy at time *s* (subsequently, a worker of "type *s*") has consumption level:

$$c_z^W(s) = \left(\frac{\gamma}{1+\tau^C}\right)\bar{h}_s e^{\sigma(z-s)} = \left(\frac{\gamma}{1+\tau^C}\right)\bar{h}_z e^{(\sigma-g+n)(z-s)} \quad \text{(A.1)}$$

The second equality follows from the fact that $\bar{h}_z$ grows at rate $g-n$ over time. A worker has consumption less than or equal to $\hat{c}_z$ if he/she entered the economy at or later than $\hat{s}$ where $\hat{s}$ solves the equation $\hat{c}_z = \frac{\gamma\bar{h}_z e^{(\sigma-g+n)(z-\hat{s})}}{1+\tau^C} = \bar{c}_z^W e^{(\sigma-g+n)(z-\hat{s})}$, where $\bar{c}_z^W = \left(\frac{\gamma}{1+\tau^C}\right)\bar{h}_z$ is the consumption of a worker entering the economy at *z*. Taking logs:

$$\hat{s} = z - \frac{\ln\left[\hat{c}_z/\bar{c}_z^W\right]}{\sigma-g+n} = z - \frac{\ln\left[\hat{c}_z/\bar{c}_z^W\right]}{p+\pi+n-\theta}. \quad \text{(A.2)}$$

The consumption level of a worker entering the economy at time *z* is the minimum level of worker consumption in the economy so $\hat{c}_z \geq \bar{c}_z^W$. The number of workers of type *s* is equal to:

$$N_z^W(s) = (p+n)e^{nz}e^{-[p+\pi+n](z-s)} \quad \text{(A.3)}$$

so consumption of all workers of type *s* at time *z* is:

$$C_z^W(s) = N_z^W(s) \cdot c_z^W(s) = \theta C_z^W e^{-\theta(z-s)} \quad \text{(A.4).}$$

Hence the combined consumption of all workers having consumption level less than $\hat{c}_z$ is

$$\begin{aligned} C_z^W(\hat{c}_z) &= \int_{s=\hat{s}}^{z} C_z^W(s) ds = \theta C_z^W \int_{s=\hat{s}}^{z} e^{-\theta(z-s)} ds \\ &= C_z^W \left(1 - e^{\frac{-\theta}{p+\pi+n} \ln\left[\hat{c}_z / \overline{c}_z^W\right]} \right) \end{aligned} \quad \text{(A.5)}$$

Letting $C_z^W(\hat{c}_z)/C_z^W = F^W(\hat{c}_z)$, equation (A.5) yields equation (16.W) in the text.

A.2 The Size Distribution of Retirees' Consumption

The consumption level of a retiree is determined by his/her social security benefit, which depends on when the person entered the economy and when the person retired. the consumption of a person who entered the economy at time *s* and retired at time *t* (henceforth, a retiree of type (*s*, *t*)) is:

$$\begin{aligned} c_z^R(s,t) &= \frac{b_z(s,t)}{1+\tau^C} = \frac{\beta \omega \overline{h}_s e^{\sigma(t-s)} e^{g^B(z-t)}}{1+\tau^C} \\ &= \overline{c}_z^R e^{[\sigma-g+n](t-s)} e^{\left[g^B-g+n\right](z-t)} \end{aligned} \quad \text{(A.6)}$$

where $\overline{c}_z^R = \frac{\beta \omega \overline{h}_z}{1+\tau^C}$ is the minimum retiree consumption level (that of a person who entered the economy and retired right away).

A retiree of type (*s*,*t*) has consumption $c_z^R(s,t) \le \hat{c}_z$ if he/she entered the economy later than $\hat{s}(t)$ where

$$\hat{s}(t) = \left(\frac{g^B - g + n}{\sigma - g + n} \right) z + \left(\frac{\sigma - g^B}{\sigma - g + n} \right) t - \frac{\ln\left[\hat{c}_z / \overline{c}_z^R\right]}{\sigma - g + n}. \quad \text{(A.7)}$$

Setting $g^B = g - n$, we obtain

$$\hat{s}(t) = t - \frac{\ln\left[\hat{c}_z / \overline{c}_z^R\right]}{\sigma - g + n} = t - \frac{\ln\left[\hat{c}_z / \overline{c}_z^R\right]}{p + \pi + n - \theta} \qquad \text{(A.7')}$$

The number of retirees of type (*s*,*t*) is $N_z^R(s,t) = (p+n)\pi e^{nz}\left(e^{-[p+\pi+n](t-s)}e^{-(n+p)(z-t)}\right)$ and the total consumption by retirees who retired at *t* and presently have consumption less than $\hat{c}_z$ is:

$$\begin{aligned} C_z^R(\hat{c}_z, t) &= \int_{s=\hat{s}(t)}^{z} N_z^R(s,t)\, c_z^R(s,t)\, ds \\ &= (p+n)\pi\overline{c}_z^R e^{nz} e^{[p+n](z-t)} \int_{s=\hat{s}(t)}^{z} e^{-\theta(t-s)}\, ds \qquad \text{(A.8)} \\ &= \frac{(p+n)\pi\overline{c}_z^R e^{nz} e^{[p+n](z-t)}}{\theta}\left(1 - e^{-\theta(t-\hat{s}(t))}\right) \end{aligned}$$

Since $\overline{c}_z^R = \frac{\beta\omega\overline{h}_z}{1+\tau^C}$ and $\overline{h}_z = \frac{\theta H_z}{(p+n)e^{nz}}$, we can write equation (A.8) as:

$$C_z^R(\hat{c}_z, t) = (p+n)C_z^R e^{[p+n](z-t)}\left(1 - e^{-\theta(t-\hat{s}(t))}\right) \qquad \text{(A.8')}$$

where $C_z^R = \frac{\beta\omega H_z}{1+\tau^C} \cdot \frac{\pi}{p+n}$ is consumption by all retirees at time *z*. Using equation (A.7'), we see that $e^{-\theta[t-\hat{s}(t)]} = e^{-\theta\left[\frac{\ln\left[\hat{c}_z/\overline{c}_z^R\right]}{p+\pi+n-\theta}\right]} = \left(\frac{\overline{c}_z^R}{\hat{c}_z}\right)^{\frac{\theta}{p+\pi+n-\theta}}$ which is independent of *t* so

$$C_z^R(\hat{c}_z) = (p+n)C_z^R\left[1 - \left(\frac{\overline{c}_z^R}{\hat{c}_z}\right)^{\frac{\theta}{p+\pi+n-\theta}}\right]\int_{t=-\infty}^{z} e^{[p+n](z-t)}dt \qquad \text{(A.9)}$$

Equation (16.R) is obtained by integrating (A.9) and letting $F^R(\hat{c}_z) = C_z^R(\hat{c}_z)/C_z^R$ denote the share of total consumption by retirees with levels less than or equal to $\hat{c}_z$.